Faculdade de Economia, Administração e Contabilidade de Ribeirão Preto
Universidade de São Paulo

Texto para Discussão

Série Economia

TD-E / 2008

Um Teste Bayesiano Para Raízes Unitárias E Co-Integração

Márcio Alves Diniz

Av. Bandeirantes, 3900 - Monte Alegre - CEP: 14040-900 - Ribeirão Preto-SP
Fone (16) 3602-4331/Fax (16) 3602-3884 - e-mail: cebelima@usp.br site:www.fearp.usp.br

Faculdade de Economia, Administração e Contabilidade de Ribeirão Preto
Universidade de São Paulo

Universidade de São Paulo

Faculdade de Economia, Administração e Contabilidade de Ribeirão Preto

A série TEXTO PARA DISCUSSÃO tem como objetivo divulgar: i) resultados de trabalhos em desenvolvimento na FEA-RP/USP; ii) trabalhos de pesquisadores de outras instituições considerados de relevância dadas as linhas de pesquisa da instituição. A série foi subdividida em função das principais áreas de atuação da FEA-RP/USP: Economia, Administração e Contabilidade. Veja o site da CPq na Home Page da FEA-RP: www.fearp.usp.br. Informações: e-mail: cpq@fearp.usp.br

FBST for Unit Root Problems

M.Diniz*,†, C.A.B.Pereira* and J.M.Stern*,†

*University of Sao Paulo, Institute of Mathematics and Statistics
†diniz_marcio@yahoo.com.br   jstern@ime.usp.br

Abstract. This paper presents the Full Bayesian Significance Test for unit roots in auto-regressive time series, and compares it to other approaches on a benchmark of 14 econometric series.
Keywords. ARMA models, e-values, FBST, Unit roots.

INTRODUCTION

Testing for unit roots in ARMA time series models is a problem that presents well known and documented difficulties for standard Bayes Factor methodologies, see [1], [2], [4], [6], [10], and [13] to [16]. In [2, p.159], the authors state:

"Testing for unit root is a Bayesian framework in one of the most controversial topics in the economic literature. There are several reasons for this:

- First ... [the use of] information that is not contained in the likelihood function and this violates the likelihood principle to which Bayesians stick.

- Secondly, the unit root hypothesis is a point hypothesis and Bayesians do not like testing point hypothesis because it is not natural to compare an interval which receives a positive probability with a point null hypothesis of zero mass.

- Finally, classical and Bayesian unit root tests do not *give the same answer. This is a striking example where it is* not *possible to recover the classical results using a non-informative prior."*

We will show that the FBST, or Full Bayesian Significance Test, presented in [9], easily overcomes all these difficulties, see also [7], and [17]. Moreover, the FBST e-values are computed following the absolutely standard form of FBST formalism, using non-informative priors, and in strict observance of the likelihood principle. Finally, the FBST analysis agrees with the classic analysis on a benchmark of 14 time series commonly used in the econometric literature.

The first section describes the FBST procedure and the numerical procedures to calculate the e-values. Afterwards, we describe the problem and the general model used to test for unit roots and derive the posterior distribution used in the present work to perform the FBST. Concluding, we present the FBST results for the mentioned time series and compare the classical and Bayesian procedures performing simulation exercises.

FBST REVIEW

The FBST was specially designed to give an epistemic value, or value of evidence, supporting a sharp hypothesis H. This support function is the *e*-value, $\text{ev}(H)$. Furthermore, the e-value has many necessary or desirable properties for a statistical support function, such as:

(I) Give an intuitive and simple measure of significance for the hypothesis in test, ideally, a *probability* defined directly in the original or *natural parameter space*.

(II) Have an intrinsically geometric definition, independent of any non-geometric aspect, like the particular parameterization of the (manifold representing the) null hypothesis being tested, or the particular coordinate system chosen for the parameter space, i.e., be an *invariant* procedure.

(III) Give a measure of significance that is smooth, i.e. *continuous and differentiable*, on the hypothesis parameters and sample statistics, under appropriate regularity conditions of the model.

(IV) Obey the *likelihood principle* , i.e., the information gathered from observations should be represented by, and only by, the likelihood function.

(V) Require *no ad hoc artifice* like assigning a positive prior probability to zero measure sets, or setting an arbitrary initial belief ratio between hypotheses.

(VI) Be a *possibilistic* support function, where the support of a logical disjunction is the maximum support among the support of the disjuncts.

(VII) Be able to provide a *consistent* test for a given sharp hypothesis.

(VIII) Be able to provide *compositionality* operations in complex models.

(IX) Be an *exact* procedure, i.e., make no use of "large sample" asymptotic approximations when computing the *e*-value.

(X) Allow the incorporation of previous experience or expert's opinion via (subjective) *prior distributions*.

The objective of this section is to provide a very short review of the FBST theoretical framework, summarizing the most important statistical properties of its support function, the *e*-value. It also summarizes the logical (algebraic) properties of the *e*-value, and its relations to other classical support calculi, including possibilistic calculus and logic, paraconsistent and classical. Further details, demonstrations of theoretical properties, comparison with other statistical tests for sharp hypotheses, and an extensive list of references can be found in the author's previous papers.

Let $\theta \in \Theta \subseteq R^p$ be a vector parameter of interest, and $L(\theta \,|\, x)$ be the likelihood associated to the observed data x, a standard statistical model. Under the Bayesian paradigm the posterior density, $p_n(\theta)$, is proportional to the product of the likelihood and a prior density, The (null) hypothesis H states that the parameter lies in the null set, defined by inequality and equality constraints given by vector functions g and h in the parameter space.

$$p_n(\theta) \propto L(\theta \,|\, X)\, p_0(\theta) \ , \ \ \Theta_H = \{\theta \in \Theta \,|\, g(\theta) \leq \mathbf{0} \wedge h(\theta) = \mathbf{0}\} \ .$$

From now on, we use a relaxed notation, writing H instead of Θ_H. We are particularly interested in sharp (precise) hypotheses, i.e., those in which $\dim(H) < \dim(\Theta)$, i.e. there is at least one equality constraint.

The FBST defines $\mathrm{ev}(H)$, the e-value, the epistemic value or value of (presented or observed) evidence supporting (in favor of) the hypothesis H, and $\overline{\mathrm{ev}}(H)$, the e-value against H, as

$$s(\theta) = \frac{p_n(\theta)}{r(\theta)} \ , \ \ s^* = s(\theta^*) = \sup_{\theta \in H} s(\theta) \ , \ \ \widehat{s} = s(\widehat{\theta}) = \sup_{\theta \in \Theta} s(\theta) \ ,$$

$$T(v) = \{\theta \in \Theta \,|\, s(\theta) \le v\} \ , \ \ W(v) = \int_{T(v)} p_n(\theta)\, d\theta \ , \ \ \mathrm{ev}(H) = W(s^*) \ ,$$

$$\overline{T}(v) = \Theta - T(v) \ , \ \ \overline{W}(v) = 1 - W(v) \ , \ \ \overline{\mathrm{ev}}(H) = \overline{W}(s^*) = 1 - \mathrm{ev}(H) \ .$$

The function $s(\theta)$ is known as the posterior surprise relative to a given reference density, $r(\theta)$. $W(v)$ is the cumulative surprise distribution. The surprise function was used, among other statisticians, by Good, Evans and Royall. Its role in the FBST is to make $\mathrm{ev}(H)$ explicitly invariant under suitable transformations on the coordinate system of the parameter space, see next section.

The tangential (to the hypothesis) set $\overline{T} = \overline{T}(s^*)$, is a Highest Relative Surprise Set (HRSS). It contains the points of the parameter space with higher surprise, relative to the reference density, than any point in the null set H. When $r(\theta) \propto 1$, the possibly improper uniform density, $\overline{T}$ is the Posterior's Highest Density Probability Set (HDPS) tangential to the null set H. Small values of $\overline{\mathrm{ev}}(H)$ indicate that the hypothesis traverses high density regions, favoring the hypothesis.

In the FBST the role of the reference density, $r(\theta)$ is to make $\overline{\mathrm{ev}}(H)$ explicitly invariant under suitable transformations of the coordinate system. Invariance, as used in statistics, is a metric concept. The reference density can be interpreted as a compact and interpretable representation for the reference metric in the original parameter space. This metric is given by the geodesic distance on the density surface. The natural choice of reference density is an uninformative prior, interpreted as a representation of no information in the parameter space, or the limit prior for no observations, or the neutral ground state for the Bayesian operation. Standard (possibly improper) uninformative priors include the uniform and maximum entropy densities.

Let us consider the cumulative distribution of the e-value against the hypothesis, $\overline{V}(c) = \Pr(\overline{\mathrm{ev}} \le c)$, given θ^0, the true value of the parameter. Under appropriate regularity conditions, for increasing sample size, $n \to \infty$, we can say the following:

- If H is false, $\theta^0 \notin H$, then $\overline{\mathrm{ev}}$ converges (in probability) to 1, that is, $\overline{V}(0 \le c < 1) \to 0$.
- If H is true, $\theta^0 \in H$, then $\overline{V}(c)$, the confidence level, is approximated by the function

$$QQ(t,h,c) = \mathrm{Q}\left(t-h, \mathrm{Q}^{-1}(t,c)\right) \ , \quad \text{where}$$

$$\mathrm{Q}(k,x) = \Gamma(k/2, x/2)/\Gamma(k/2,\infty) \ , \ \ \Gamma(k,x) = \int_0^x y^{k-1} e^{-y} dy \ ,$$

$t = \dim(\Theta)$, $h = \dim(H)$ and $\mathrm{Q}(k,x)$ is the cumulative chi-square distribution with k degrees of freedom.

Under the same regularity conditions, an appropriate choice of threshold or critical level, $c(n)$, provides a consistent test, τ_c , that rejects the hypothesis if $\overline{\mathrm{ev}}(H) > c$.

The empirical power analysis developed in [7] and [18], provides critical levels that are consistent and also effective for small samples.

THE AUTO REGRESSIVE TIME SERIES MODEL

The $AR(1)$ process

$$y_t = \phi y_{t-1} + \varepsilon_t$$

where $\varepsilon_t \sim i.i.d.(0, \sigma^2)$, has a unit root if $\phi = 1$. In this case its mean and its variance do not exist. If $|\phi| < 1$, then the mean of y_t is zero and its variance $\sigma^2/(1-\phi^2)$ and the process has a strong tendency to return to its mean value after a shock. However, if the process has a unit root, a shock has an everlasting effect. This can be seen if y_t is expressed as the cumulated sum of past errors, each with the same weight. Therefore, test for a unit root consists in testing the precise hypothesis $H_0 : \phi = 1$.

The economic and econometric literature has given great importance to the development of unit root tests in the past two and a half decades. It is very important to know if, for instance, economic recessions have permanent consequences for the level of future GNP, or instead represent just a temporary downturn with the output lost eventually made up during recovery. Nelson and Plosser, [10], argued that many economic series are better characterized by unit roots than by deterministic trends.

However, in the development of the tests difficulties arised because the asymptotic distribution of the ordinary least squares estimators presents a discontinuity at $\phi = 1$. The ADF test is the most used in unit root tests and assumes, in its more general form, that the data generating process has a constant, a deterministic trend and follows an $AR(p)$ structure with $i.i.d.$ errors. Below we introduce this model assuming gaussian disturbances to develop the bayesian inference.

The $AR(p)$, or order p auto-regressive time series model with white Gaussian noise and deterministic intercept and trend, is written as:

$$y_t = \mu + \delta\, t + \phi_1 y_{t-1} + \ldots + \phi_p y_{t-p} + \varepsilon_t$$

where $\varepsilon_t \sim N(0, \sigma^2)$ $\forall t = 1, \ldots, T$. This series can also be written in the differenced or correction form:

$$\Delta y_t = \mu + \delta\, t + \Gamma_0 y_{t-1} + \Gamma_1 \Delta y_{t-1} + \ldots + \Gamma_{p-1} \Delta y_{t-p+1} + \varepsilon_t$$

where $\Delta y_t = y_t - y_{t-1}$, $\Gamma_0 = \phi_1 + \ldots + \phi_p - 1$ and $\Gamma_i = -\sum_{j=i+1}^{p} \phi_j$, for $i = 1, \ldots, p-1$.

If using this parametrization, the series has a unit root if $\Gamma_0 = 0$. The ADF tests this hypothesis against $\Gamma_0 \leq 0$, but if $\Gamma_0 \geq 0$ the process is non-stationary.

This model can also be written in standard regression form, using the parameter vector $\theta = [\beta, \sigma]$, where β is a vector with all the linear parameters, $Y_p = [y_1 \ldots y_p]$ is the vector of the first p observations, and Y is the vector of all remaining observations:

$$Y = X\beta + e \text{ ,where}$$

$$\beta = \begin{bmatrix} \mu \\ \delta \\ \Gamma_0 \\ \Gamma_1 \\ \ldots \\ \Gamma_{p-1} \end{bmatrix} , \quad Y = \begin{bmatrix} \Delta y_{p+1} \\ \Delta y_{p+2} \\ \ldots \\ \Delta y_T \end{bmatrix} , \quad X = \begin{bmatrix} 1 & 1 & y_p & \Delta y_p & \ldots & \Delta y_2 \\ 1 & 2 & y_{p+1} & \Delta y_{p+1} & \ldots & \Delta y_3 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 1 & T & y_{T-1} & \Delta y_{T-1} & \ldots & \Delta y_{T-p+1} \end{bmatrix}$$

The dimensions of these matrices are, respectively, $p+2\times 1$ for β, $T-p\times 1$ for Y, and $T-p\times p+2$ for X.

Using the matrix regression form, it is easy to see that the ML estimator of β, the predicted ML observations, and the sum of squared errors is given by

$$\widehat{\beta}=(X'X)^{-1}X'Y\ ,\ \ \widehat{Y}=X\widehat{\beta}\ ,\ \text{and}$$

$$e'e=(Y-X\beta)'(Y-X\beta)=(Y-\widehat{Y})'(Y-\widehat{Y})+(\beta-\widehat{\beta})'X'X(\beta-\widehat{\beta})\ .$$

Using the standard non-informative prior $f(\beta,\sigma)\propto 1/\sigma$, the model posterior can be written as:

$$f(\beta,\sigma|Y,Y_p)\propto\sigma^{-(T-p+1)}\exp\left(-\frac{1}{2\sigma^2}\left((Y-\widehat{Y})'(Y-\widehat{Y})+(\theta-\widehat{\theta})'X'X(\theta-\widehat{\theta})\right)\right)$$

NUMERICAL EXPERIMENTS AND RESULTS

After the model derived above we tested for unit roots 14 U.S. macroeconomic time series first mentioned in Nelson and Plosser, [10]. Here we use the extended series, used in Schotman and van Dijk, [14].

The following table shows the e-values and ADF test for the aforementioned econometric time series. The ADF, Augmented Dickey and Fuller test, based on the Frisch-Waugh-Lovell theorem, is arguably the most used unit root test in econometrics. We have used the computer procedure described in James MacKinnon, at Queen's University, [8]. All numerical time series follow the specification in Bauwens et al. [2], so that the results are comparable.

As can be seen from the posterior expression, the conditional posteriors are $\pi(\theta|\sigma,Y,Y_p)\propto N(\widehat{\theta},\sigma^2V)$ and $\pi(1/\sigma^2|\theta,Y,Y_p)\propto\Gamma\left(\frac{T-p+3}{2},B\right)$, where $B=0.5(Y-\widehat{Y})'(Y-\widehat{Y})+(\theta-\widehat{\theta})'X'X(\theta-\widehat{\theta})$ and $V=(X'X)^{-1}$. For the FBST computations, several solvers can be used in the optimization step, as [3] or [5], and standard Monte Carlo sampling is used in the integration step, see [7].

In table 1 we can see that the non-stationary posterior probability is quite distant from the ADF p-value. These results were highlighted by Sims, [15] and Sims and Uhlig, [16]. Considering the simplest $AR(1)$ model, they argued that, once classical inference is based on the distribution of $\widehat{\phi}|\phi=1$, it reaches counterintuitive conclusions because the referred distribution is skewed. Bayesian inference, they conclude, uses the distribution of $\phi|\widehat{\phi},y_1\ldots,y_T$ which is not skewed.

Phillips, [13] claims that the difference in results between classical and bayesian approaches is due to the flat prior that puts much weight on the stationary region. He proposed the use of Jeffreys priors, which restored the conclusions drawn by the classical test. Phillips argued that the flat prior was, actually, informative when used in time series models like those for unit root tests. He made simulations that show the

" [the use of a] flat prior has a tendency to bias the posterior towards stationarity. ... even when [the estimator] is close to unity, there may still be a non negligible downward bias in the [flat] posterior probabilities".

TABLE 1. Unit root tests for Nelson and Plosser data

Series	start	p	trend	ADF	p-value	$P(\Gamma_0 \geq 0 \vert Y)$	e-value
Real GNP	1909	2	yes	-3.52	0.044	0.0005	0.040
Nominal GNP	1909	2	yes	-2.06	0.559	0.0238	0.523
Real GNP per capita	1909	2	yes	-3.59	0.037	0.0004	0.034
Industrial prod.	1860	2	yes	-3.62	0.032	0.0003	0.028
Employment	1890	2	yes	-3.47	0.048	0.0004	0.043
Unemployment rate	1890	4	no	-4.04	0.019	0.0001	0.020
GNP deflator	1889	2	yes	-1.62	0.778	0.0584	0.762
Consumer prices	1860	4	yes	-1.22	0.902	0.1154	0.983
Nominal wages	1900	2	yes	-2.40	0.377	0.0106	0.341
Real wages	1900	2	yes	-1.71	0.739	0.0475	0.715
Money stock	1889	2	yes	-2.91	0.164	0.0029	0.147
Velocity	1869	2	yes	-1.62	0.779	0.0620	0.777
Bond yield	1900	4	no	-1.35	0.602	0.0962	0.936
Stock prices	1871	2	yes	-2.44	0.357	0.0103	0.349

TABLE 2. MLE under $H_0 : \Gamma_0 = 0$

Parameters	Real GNP	Ind. Prod.	GNP def.	Wage
μ	0.01543	0.049427	0.00187	0.01494
δ	0.00011	-0.00014	0.00027	0.00020
Γ_1	0.33146	0.03636	0.44992	0.46687
σ	0.05558	0.09682	0.04364	0.05545

TABLE 3. Standard error of MLE under $H_0 : \Gamma_0 = 0$

Parameters	Real GNP	Ind. Prod.	GNP def.	Wage
μ	0.01320	0.01806	0.00902	0.01247
δ	0.00028	0.00024	0.00016	0.00024
Γ_1	0.10895	0.08966	0.09163	0.09661

TABLE 4. MLE - unrestricted model

Parameters	Real GNP	Ind. Prod.	GNP def.	Wage
μ	0.81849	0.05221	0.09086	0.39792
δ	0.00567	0.00718	0.00112	0.00309
Γ_0	-0.17631	-0.17658	-0.03164	-0.06494
Γ_1	0.41106	0.12432	0.46979	0.50130
σ	0.05193	0.09252	0.04329	0.05392

Tables 2 and 3 display some ML estimators and the respective standard errors assuming unit roots. Table 4 and 5 show the ML estimators for the same series for the unrestricted model. Table 6 and 7 give the number of series which rejected the unit root hypothesis in 100 generated samples assuming that there was (table 6) or not (table 7) a unit root. We used three criteria to reject the hypothesis: the ADF asymptotic p-value for 5% significance, the exact ADF p-value for 5% significance and the e-value set in 0.05.

It is important to remember that finite sample critical values for unit root tests depend

TABLE 5. Standard error of MLE - unrestricted model

Parameters	Real GNP	Ind. Prod.	GNP def.	Wage
μ	0.23279	0.01727	0.05667	0.16301
δ	0.00163	0.00206	0.00056	0.00125
Γ_0	0.05104	0.04941	0.01990	0.02756
Γ_1	0.10436	0.08915	0.09175	0.09522

TABLE 6. Simulated series rejecting H_0 in hundred generated assuming H_0

Series	$< ADF_{5\%}(\infty)$	$< ADF_{5\%}(ex.)$	$ev < 0.05$
Real GNP	4	3	3
Ind. Prod.	4	4	4
GNP def.	7	6	6
Wage	4	4	4

TABLE 7. Simulated series rejecting H_0 in hundred generated assuming the unrestricted model

Series	$< ADF_{5\%}(\infty)$	$< ADF_{5\%}(ex.)$	$ev < 0.05$
Real GNP	73	67	64
Ind. Prod.	85	82	84
GNP def.	20	18	18
Wage	29	27	27

on the assumption that the error terms are $N(0, \sigma^2 I)$ once these values were generated by simulations that use this assumption. The asymptotic critical values are valid much more generally, since they do not require normality or homoskedasticity. Therefore, for small samples, it is safer to rely on asymptotic critical values.

Table 6 shows that the FBST, even using the flat prior, has a power similar to the ADF test. Hence, the argument used by Phillips to criticize conclusions based on posterior probabilities when flat priors were used is not valid for the FBST.

We perform more numerical simulations to compare the ADF and the FBST powers. The exercise was the following. After simulating 1000 series with the data generator processes:

$$y_t = \phi y_{t-1} + \varepsilon_t \tag{1}$$

$$y_t = \mu + \phi y_{t-1} + \varepsilon_t \tag{2}$$

$$y_t = \mu + \delta t + \phi y_{t-1} + \varepsilon_t, \tag{3}$$

we calculate the ADF statistic and the e-value. In order to reject or not the hypothesis tested we used the ADF 5% significance level given for samples with size of 50 observations. For the FBST we used the level one defined empirically, i.e. the e-value for the 5% percentile when the hypothesis is true. The tables below summarize the results.

TABLE 8. Number of times in which $H_0 : \phi = 1$ was rejected for the 1000 series generated by (1)

Parameters	ADF	$Ev < 0.144$
ϕ=1	42	50
ϕ=0.99	65	63
ϕ=0.975	83	84
ϕ=0.95	139	132

TABLE 9. Number of times in which $H_0 : \phi = 1$ was rejected for the 1000 series generated by (2) - $\mu = 0.5$

Parameters	ADF	$Ev < 0.15$
ϕ=1	10	50
ϕ=0.99	33	101
ϕ=0.975	65	188
ϕ=0.95	110	300

TABLE 10. Number of times in which $H_0 : \phi = 1$ was rejected for the 1000 series generated by (3) - $\mu = 0.5$, $\delta = 0.02$

Parameters	ADF	$Ev < 0.0654$
ϕ=1	13	50
ϕ=0.99	46	111
ϕ=0.975	55	133
ϕ=0.95	55	167

The ADF and FBST have similar power for the model without deterministic terms. For the models with constant and with constant and deterministic trend, the FBST has a better performance even if we consider the statistic asymptotic levels.

Another great advantage of the FBST is the possibility to perform the test even if the data set does not have gaussian distribution. Bearing this in mind, we present another exercise. We simulated 1000 series with the random term following a Gumbel distribution with nil location parameter and unitary scale parameter. Below we present the standard normal and the Gumbel(0,1) densites. We used the simplest AR(1) DGP for this exercise. Table 11 summarizes the results.

Once the ADF statistic critical levels were calculated by simulations assuming the gaussian distribution of the error term, conclusions based on this test are compromised when one is not sure about the data set normality, specially for small samples. Even using the asymptotic critic level for the ADF statistic, as proposed above, the conclusions do not change since for $T = 50$ the critic level for 5% is -1.9475 and the asymptotic is -1.9408.

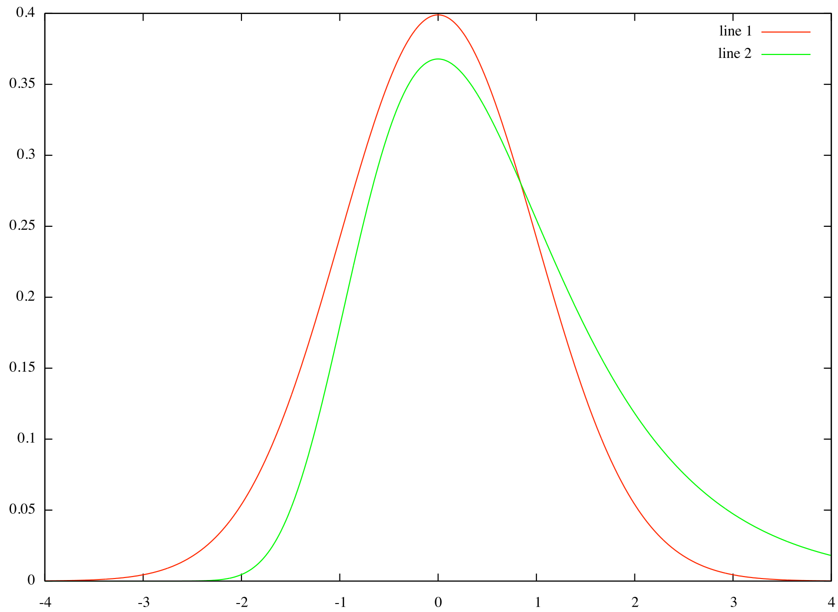

FIGURE 1. Line 1 - Normal (0,1) and Line 2 - Gumbel (0,1)

TABLE 11. Number of times in which $H_0 : \phi = 1$ was rejected for the 1000 series generated by (1) with Gumbel(0,1) error.

Parameters	ADF	$Ev < 0.14$
ϕ=1	0	50
ϕ=0.99	0	80
ϕ=0.975	0	176
ϕ=0.95	0	339

CONCLUDING REMARKS

As mentioned in the first section, Bayes Factor tests for unit roots have had many difficulties to deal with time series presented in the field of econometrics. Several alternative Bayes Factor tests have been proposed in order to overcome these difficulties. However, their performance is still in question. For example, [1] concludes:

"In two Monte Carlo simulations, however, we find that the 'objective' Bayesian test have relatively low power in distinguishing between plausible alternatives, making it difficult to draw any conclusions concerning long-run [performance]. We conclude that, at least for the 'objective' Bayesian test, the Bayesian approach is not necessary better than the classical ADF approach."

Based on simulation studies, [6] suggests that practitioners must assign a high probability to the value to be tested in order to get high power when using Bayes Factor tests, although this means to increase the non-stationary weight when testing for unit root.

There have also been other tests based on or using specially designed priors, that show a better performance. However, the use of such priors departs from some basic paradigms of Bayesian statistics, like the Likelihood Principle. Moreover, these techniques have to be fine tuned to each particular problem type or application. In contrast, the FBST e-value derivation and implementation is straightforward from its general definition, using absolutely no ad hoc artifice, like a special prior, or a measure on the hypothesis set induced by some special parameterization, or an arbitrary initial likelihood ratio. Hence, the FBST is in strict compliance with Likelihood Principle. Moreover, it can be used when the normality assumption is not verified and for this the researcher only has to choose another parametric or semi-parametric family in order to derive the likelihood and the posterior.

REFERENCES

1. F.W.Ahking (2004). The Power of the "Objective" Bayesian Unit-Root Test. University of Connecticut Department of Economics Working Paper 2004-14.
2. L.Bauwens, M.Lubrano, J.F.Richard (1999). *Bayesian Inference in Dynamic Econometric Models.* Oxford University Press.
3. E.G.Birgin, R.Castillo, J.M.Martinez (2004). Numerical Comparison of Augmented Lagrangian Algorithms for Nonconvex Problems. *Computational Optimization and Applications,* 31, 1-56. Software and documentation available at *www.ime.usp.br/∼egbirgin/tango/* .
4. C.Conigliani, F.Spezzaferri (2002). A Robust Bayesian Approach for Unit Root Testing. To appear in *Econometric Theory*, 23, 3, 2007.
5. W.L.Goffe, G.D.Ferrier, J.Rogers (1994). Global Optimization of Statistical Functions with Simulated Annealing. *Journal of Econometrics*, 60, 65–99.
6. Z. I. Kalaylioglu (2002). *Frequentist and Bayesian Unit Root Tests in Stochastic Volatility Models*, PhD Thesis, NC State University.
7. M.Lauretto, C.A.B.Pereira, J.M.Stern, S.Zacks (2003). Full Bayesian Significance Test Applied to Multivariate Normal Structure Models. Comparing Parameters of Two Bivariate Normal Distributions Using the Invariant Full Bayesian Significance Test. *Brazilian J. Probability and Statistics,* 17, 147-168.
8. J.G.MacKinnon (1994). Approximate Asymptotic Distribution Functions for Unit-Root and Cointegration Tests. *Journal of Business and Economic Statistics*, 12, 167-176.
9. M.R.Madruga, C.A.B.Pereira, J.M.Stern (2003). Bayesian Evidence Test for Precise Hypotheses. *Journal of Statistical Planning and Inference*, 117,185–198.
10. C.R.Nelson, C.I.Plosser (1982). Trends and Random Walks in Macroeconomics Time Series: Some Evidence and Implications. *Journal of Monetary Economics*, 10, 139–162.
11. C.A.B.Pereira, J.M.Stern, (1999). Evidence and Credibility: Full Bayesian Significance Test for Precise Hypotheses. *Entropy Journal*, 1, 69–80.
12. C.A.B.Pereira, J.M.Stern (2001). Model Selection: Full Bayesian Approach. *Environmetrics* 12, 6, 559-568.
13. P.C.B.Phillips (1991). To Criticize the Critics: An Objective Bayesian Analysis of Stochastic Trends. *Journal of Applied Econometrics*, 6, 333–364.
14. P.C.Schotman, H.K. van Dijk (1993). Posterior Analysis of Possibly Integrated Time Series with an Application to Real GNP. in P.Caines, J.Geweke, M.Taqqu (eds.), *New Directions in Time Series Analysis, Part II.* Springer-Verlag, New York.
15. C.A.Sims (1988). Bayesian Skepticism on Unit Root Econometrics. *Journal of Economics Dynamic and Control*, 12, 463–474.
16. C.A.Sims, H.Uhlig (1991). Understanding Unit Rooters: A Helicopter Tour. *Econometrica*, 59, 1591-1600.

17. J.M.Stern (2007). Cognitive Constructivism, Eigen-Solutions, and Sharp Statistical Hypotheses. *Cybernetics and Human Knowing,* 14,1, 9-36.
18. J.M.Stern, S.Zacks (2002). Testing Independence of Poisson Variates under the Holgate Bivariate Distribution. The Power of a New Evidence Test. *Statistical and Probability Letters*, 60, 313–320.