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New Light on von Neumann: politics, psychology and the creation of game theory*

Robert Leonard**

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Introduction

> "Dear, my dislike for Europe has nothing to do with Mariette. I feel the opposite of a nostalgia for Europe, because every corner I knew reminds me of the world, of the society, of the excitingly nebulous expectations of my childhood . . . of a world which is gone, and the ruins of which are no solace. . . My second reason for disliking Europe is the memory of the my total disillusionment in human decency between 1933 and September 1938, the advent of Nazism and the reaction of humanity to it – in that period I suffered my life's greatest emotional shock…"[1]

Thus wrote John von Neumann to his wife Klari, in the summer of 1949, on the stationery of the Hôtel Ruhl, Nice, but probably when already back in Princeton. He was writing in apology for he had just abandoned her on the French Riviera, making a bee-line back to the U.S. The European trip had been von Neumann's first since the end of the war, reluctantly undertaken under pressure from Klari, his second wife – "Mariette" was his first – who had not returned to the Continent since their marriage in 1938.[2] To please her, he had grudgingly accepted to speak at an astrophysics

[1]John von Neumann (Hôtel Ruhl, Nice) to Klari von Neumann, Thursday (no date), 1949, located in the papers of the late Klari von Neumann-Eckhart, which are in the possession of John von Neumann's daughter, Professor Marina von Neumann-Whitman of Ann Arbor, Michigan, and which I shall hereafter refer to as KEMNW.

[2] See Klari von Neumann-Eckhart (no date) "Johnny", draft chapter of an unpublished autobiography "Grasshopper in the Long Grass", KEMNW. This was written sometime between von Neumann's death in 1957 and Klari's own death, by suicide, in 1963.

conference in Paris. Then, on the way to Capri, deciding he could stand it no longer, he fled. "I was worried", he explained, "when I saw, what a trip to Capri, to recapture the past, was beginning to mean to you".

> "After all, the past cannot be truly recaptured, it is elusive, it must have an unspecific, dreamlike quality, and to try to distill it, to precipitate it, to crystallize it, is courting disappointment.
>
> I got more worried, when you continued by tying up the Capri venture with a complicated and elaborate European journey, knowing that you stand running around and changing habitats, etc. badly. It was even more dangerous to combine this with family matters, knowing the quality of those… The past is hard enough to resuscitate "in camera", in the deepest and safest recess of your own lower consciousness – but how can you, for this descent into the underworld, team up with others, who have since had other pasts, who have their own interests or rainbows to chase, and who do not have the quality of your dreams and emotions, and who are not your equals[?]…
>
> And lastly, there was a phenomenal piece of foolhardy courage in taking me along for this ride – a phenomenal compliment that I appreciated as soon as I understood it for what it was – but very, very dangerous nevertheless. We have riskily uncertain "communications" even on normal matters, and in this case you had as valid and violent reasons for wanting to find the holy grail, as I was undesirous to lay eyes on it"[3]

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[3] Von Neumann to Klari, op cit.

Insofar as it was taken up with reassuring a psychologically frail Klari, the letter is like many von Neumann wrote to her over the course of their often tempestuous relationship. Less typical, however, is the manner in which it speaks to von Neumann's own anxieties.

To the historian of science, von Neumann is known as one of the century's most effective mathematicians, of remarkable accomplishment in a range of pure mathematical fields as well as in quantum mechanics, atomic physics, computer science and game theory. To the layman he is known variously for his importance at Los Alamos, his notoriety as Cold War "hawk", and perhaps even his partial inspiration of Stanley Kubrick's Dr. Strangelove. During World War II, von Neumann was one of the U.S.'s most cherished military advisors, and this was followed by deep postwar involvements in the RAND Corporation and the Atomic Energy Commission. He is not typically associated with emotional weakness or existential anxiety, much less with the evocation of dreams, the underworld and the Holy Grail. And, yet, when, after his death in 1957, Klari sought to capture him in words, she wrote first of his complexity: "I want to tell you about the man, the strange, contradictory and controversial person; childish and good-humored, sophisticated and savage, brilliantly clever yet with very limited, almost primitive lack of ability to handle his emotions – an enigma of nature that will have to remain unsolved".[4] In short, beneath the resilient façade of the public scientist lay a multi-faceted personality, the probing of which will be part of the task of this paper.[5]

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[4] Klari von Neumann, "Johnny", p. 3.

[5] On von Neumann, see Steve Heims, *John von Neumann and Norbert Wiener: From Mathematics to the Technologies of Life and Death* (Cambridge, Mass.: MIT Press, 1980) and Norman Macrae,

Given the prominence of von Neumann's contributions to more central areas of mathematics and science, to say nothing of the atomic bomb, his work in game theory has tended to be viewed as something of a side-interest, a minor foray into the realm of social science. Yet this essay will argue that there was nothing remotely peripheral about game theory as far as von Neumann was concerned. Precisely because it was concerned with the nature of rational behaviour and the representation of social structure, game theory went to the very heart of who von Neumann was, as a Hungarian, Jewish, émigré mathematician. Of all his scientific activities, game theory bore a particularly intimate relationship to his "life".

John von Neumann: The Scientific Genius who pioneered the Modern Computer, Game Theory, Nuclear Deterrence and Much More (New York: Pantheon Books Macrae, 1992). Heims is critical of the Cold War hawk. Macrae celebrates "Johnny's" scientific genius and anti-communism. Neither devotes significant attention to the creation of game theory. In Philip Mirowski, *Machine Dreams: Economics becomes a cyborg science (*New York & Cambridge: Cambridge University Press, 2002) the treatment of von Neumann is inflected by his later work on computing and automata. The short essays by Marina von Neumann-Whitman, Françoise Ulam, Peter Lax and others in James Glimm, John Impagliazzo and Isadore Singer (eds.) *Proceedings of Symposia in Pure Mathematics*, Vol. 50: *The Legacy of John von Neumann*, (Providence: Americam Mathematical Society, 1990), are balanced and informative, as are the discussions in William Aspray, Péter Horváth, Dénes Nagy, Edward Teller, Nicholas Vonneumann and Eugene P. Wigner "Discussion: John von Neumann – A Case Study of Scientific Creativity", *Annals of the History of Computing*, 1989, *11*:165 -169. Amongst the popular treatments, see especially William Poundstone's *Prisoner's Dilemma* (New York: Anchor, 1992).

Von Neumann came from a prosperous and cultivated assimilated Jewish family in Budapest. As a mathematical "*Wunderkind*" in Germany in the 1920's, he inhabited a scientific universe that centred upon mathematicians Klein, Hilbert and Courant at Göttingen and embraced Planck and Einstein in Berlin, Weyl and Schrödinger in Zurich, and the Bohrs in Copenhagen. It was a world of scientific plenty, a "near-paradise for the academically minded", Klari would later say, to which von Neumann would long remain attached.[6] It was here, at Göttingen, in this *Wunderkind* phase, that the 23-year old first concerned himself with the mathematics of games, producing a brilliant theorem on the generic two-person, zero-sum game, the underlying inspiration for which came from the consideration of chess and cards. He published this paper in *Mathematische Annalen* in 1928, and then put the subject completely aside.[7]

As of 1933, von Neumann's world began to unravel. From his new vantage point in the U.S., he watched the dissolution of that paradise in which he had come of age, with the Nazis' destruction of a great part of German mathematics and science, and the forced exile of Jewish scientists. In relentless detail, he followed the descent into war under Hitler, and watched the destabilization, and later obliteration, of the social order of which he was product. "No wonder that his hatred, his

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[6] Klari, "Johnny", p.11.

[7] John von Neumann, "Zur Theorie der Gesellschaftsspiele", *Mathematische Annalen, 100:*pp. 295-320; trans. by S. Bargmann as "On the Theory of Games of Strategy", *Contributions to the theory of games*, Vol. 4, Eds. Albert Tucker and R. Duncan Luce (Princeton: Princeton U. Press, 1959), pp. 13-42.

loathing for the Nazis was essentially boundless", recalled Klari, "they came and destroyed the world of this perfect intellectual setting".[8] It was during the traumatic period at the end of the 1930's, at a threshold in his scientific life, that von Neumann, after a 10-year hiatus, returned to game theory. He extended his analysis from the mathematics of two-person parlour games to that of social interaction proper, publishing all in the 1944 *Theory of Games and Economic Behavior*, co-authored with Viennese economist exile, Oskar Morgenstern.[9]

Our story proceeds as follows. Part I provides essential background on the Hungarian Jewish and mathematical communities from which von Neumann came. Part II considers his initial foray into games in the 1920's, in a context marked by a great cultural interest in chess. Part III takes us closer to von Neumann as a person, tracing his engagement with the events of the 1930's and his convergence, in that context, upon a new mathematics of society. His creation of game theory appears as an intensely personal creative act, undertaken at a time of difficulty, and perhaps even serving a therapeutic purpose for the man himself.

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[8] Klari, "Johnny", p. 12.

[9] John von Neumann and Oskar Morgenstern, *Theory of Games and Economic Behavior* (Princeton: Princeton University Press, 1947, 1944). See also John von Neumann and Oskar Morgenstern, *Theory of Games and Economic Behavior*, 60th anniversary edition, with an introduction by H. Kuhn and an afterword by A. Rubinstein (Princeton: Princeton University Press, 2004).

Part I: The Hungarian Background

"Deeply rooted, yet alien"

Most histories of the Jews of Hungary emphasize the degree to which, beginning in the mid-19th century, they achieved integration into Hungarian society.[10] 1867 was the year of the *Ausgleich*, or Compromise, when the Hapsburg Monarchy, in the face of nationalist pressure, granted greater autonomy to Hungary. Law XVII of that year, on the "emancipation of the inhabitants of the Israelite faith of the country", allowed Jews to hold various commercial licenses, practice certain professions and enter parts of the public service. This marked the beginning of a flourishing period for Hungary, accompanied by the assimilation of many Jews into the economic and cultural life of the country. From the 1870's onwards, assimilation was greatest amongst the less religiously strict Neolog Jews, amongst whom it became quite common, for example, to educate children at non-Jewish schools, change one's surname in favour of a more Hungarian-sounding one, and even go so far as to choose Christian baptism. By the late 19th century, quite a few Jewish businessmen and professionals were awarded titles of nobility for their services to the Austro-Hungarian Empire.

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[10] See Randolph L. Braham, *The Politics of Genocide: the Holocaust in Hungary*, Vol. I. (New York: Columbia University Press, 1981); Nathaniel Katzburg, *Hungary and the Jews: Policy and Legislation, 1920-1943* (Ramat-Gan: Bar-Ilan University Press, 1981); Ralph Patai, *The Jews of Hungary: history, culture, psychology* (Detroit: Wayne State University Press, 1996) and Kinga Frojimovic, Géza Komoróczy, Viktória Pusztai and Andrea Strbik, *Jewish Budapest, Monuments, Rites, History* (Budapest: Central European University Press, 1999).

Banker, Miksa (Max) Neumann, the mathematician's father, was one such person, acquiring the title "von Margittai Neumann" from the emperor Franz Joseph in 1913.[11]

The von Neumann family was part of a merchant and financial community that, although of Jewish origin, saw itself as patriotically Hungarian. Jansci (John) von Neumann was educated, not at Hebrew school, but at the Lutheran Gymnasium, along with Eugene Wigner, later a physicist, and William Fellner, who became an economist. Like Theodore von Karman and Edward Teller at the Minta Gymnasium, these assimilated Jews were conscious of their cultural inheritance, yet felt themselves to be Hungarian through and through.[12] The first suggestion that this might no longer be

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[11] Max Neumann (1870 – 1923) had arrived in Budapest at age 10 from Pecs in the Southwest. Trained as a lawyer, he married Margit Kann, daughter of Jacob Kann, whose fortune had been made selling agricultural equipment and hardware to Hungary's large farms. The Kann-Heller firm was located on the ground floor of 62 Vaczi Boulevard (later renamed Bajcsy-Zsilinszky St.), with the rest of the building being divided into apartments. The Hellers occupied the second floor and the Kanns the remaining two. The top floor, 18-room apartment went to Max and Margit, to whom John von Neumann was born. See Macrae, *John von Neumann* (cit. n.5), pp. 37-46.

[12] From a similar background came banker's son and Communist, György (George) Lukács (1885-1971): "The Leopoldstadt families were completely indifferent to all religious matters. Religion only interested us as a matter of family convention, since it played a certain role at weddings and other ceremonies … we all regarded the Jewish faith with complete indifference", György Lukács, *Record of a life* (London: Verso, 1983), p. 26. "At the Protestant Gymnasium I attended, children from Leopoldstadt played the role of the aristocracy. So I was regarded as a Leopoldstadt

possible appeared at the end of World War I, when John von Neumann was in his teens, and old Hungary was broken up.

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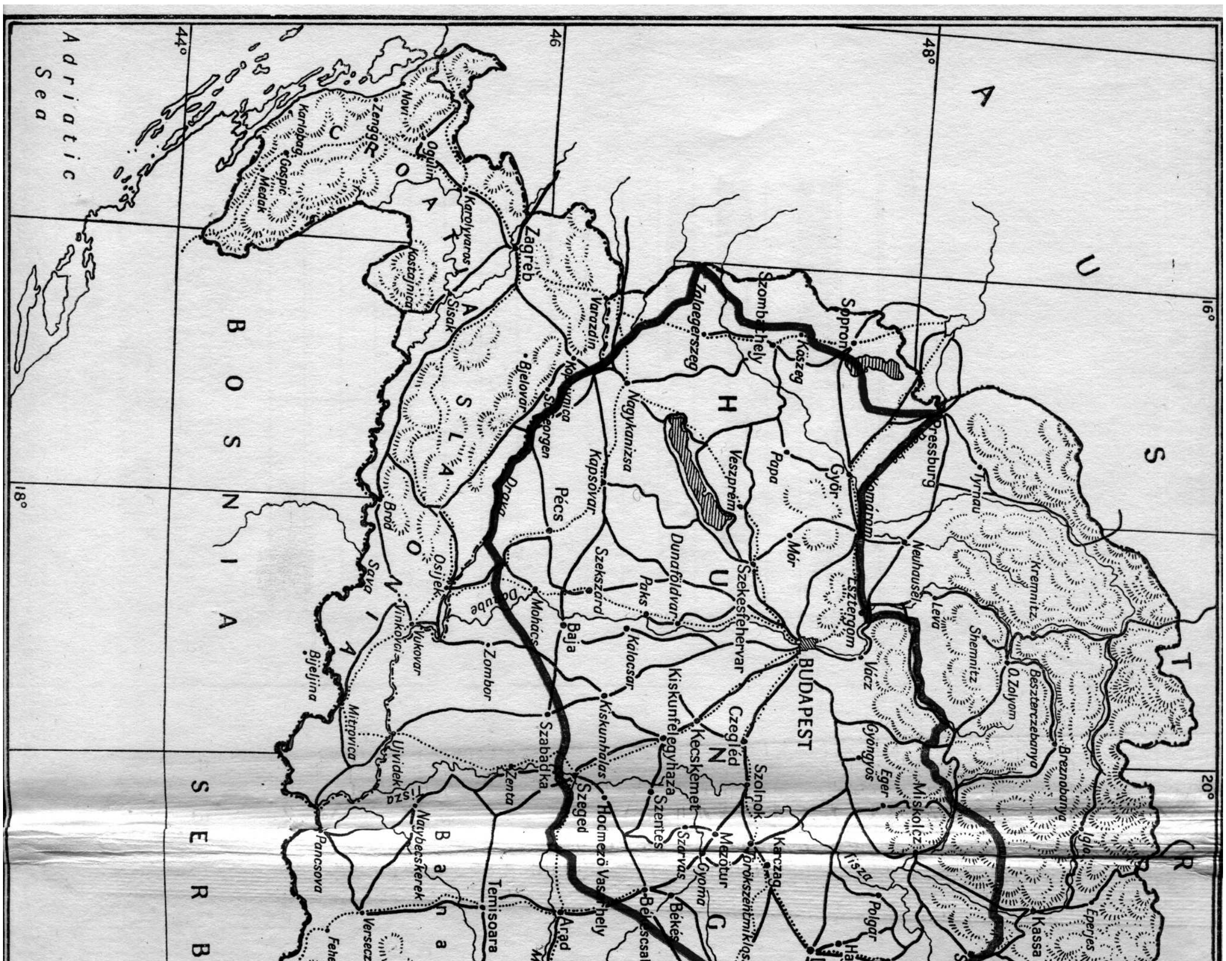
aristocrat, not as a Jew. Hence the problems of the Jews never came to the surface. I always realized that I was a Jew, but it never had a significant influence on my development" Lukács, *Record of a life*, p. 29. Von Neumann's brother, Nicholas, recalls their other brother, Michael, questioning the family's ambiguous religious stance, to which Max von Neumann replied that it was simply a matter of tradition. Stan Ulam, a Polish Jew and close friend of von Neumann, recalled that the tradition of talmudistic Judaic scholarship was "quite conspicuously absent from von Neumann's makeup" , but he remembered his indulging in Jewish jokes and banter. "The goys have the following theorem...". Stan M. Ulam, *Adventures of a Mathematician* (New York: Charles Scribner's Sons, 1976), on p. 97.


AUSTRIA
GALICIA
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SERBIA
BOSNIA
CROATIA
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Adriatic Sea
BUDAPEST
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Pre 1914 international boundary of Hungarian Crown Lands
Post 1919 boundary of Hungary
Railways
Roads
JORDAN

In 1920, the Treaty of Trianon saw the dismantling of the Austro-Hungarian empire, with Hungary required to sacrifice no less than two-thirds of its lands, and, with them, one-third of ethnic Hungarians, to the successor states, Czeckoslovakia, Rumania, Yugoslavia and Austria. The result was a greatly reduced "rump Hungary", and the birth of revisionist ambitions to regain the lost territories.

After Trianon, the position of the Hungarian Jews began to change. With the disappearance of other large ethnic groups with the surrendered regions, the integrated Jews lost part of their political function in the Jewish-Magyar alliance. The numbers of Jews in the country were swelled by further immigrants from the east, many of them Orthodox, keen, as they were, to remain within Hungary. The result was a sharpening of focus on Hungarian Jewry, with all the usual contradictions inherent in such scapegoating. For example, even if most assimilated Jews were opposed to Bela Kun's brief Communist insurgency in the Summer of 1919, the fact that a majority of the revolutionary Commissars were Jewish contributed to the popular image of the "Jewish Bolshevik". Thus when Admiral Nikolas Horthy regained power and cracked down on Kun's supporters in the White Terror of 1920, a great many of those persecuted or forced to flee were Jewish. At the same time, the visible Jewish presence in commercial life, coupled with the extravagant display of riches of a few, served to reinforce the popular perception of enormous wealth. It was in this context that Horthy's Hungary, in 1920, passed the first piece of anti-Semitic legislation in 20th century Europe. Ostensibly designed to control university registrations in general, the key clause of Law 1920: XXV, the *Numerus Clausus*, was one intended to restrict Jewish access to higher education, and therefore the professions, to a level corresponding to their proportion of the population.

Thus while their merchant and banking parents had flourished during the Golden Age following emancipation, the more highly-educated generation of Jewish youth that matured around the time of World War I were to be less settled. Not only was Hungary already small and limited in terms of opportunities in science and education; there was also the added discriminatory element. By the time the 1920 law was passed, many young Hungarian Jews had either already left or had begun looking abroad for opportunities. Prominent examples include George Pólya, Michael Polanyi, and Theodor von Kármán.[13] Although his exceptional abilities would have secured him a position within any quota, John von Neumann became part of a student diaspora, several thousand strong, in Austria, Czeckoslovakia, Germany, Italy or Switzerland. In contrast to the desultory university environment they were leaving behind, these students were enthusiastically received abroad, especially by the university mandarins in the tolerant climate of Weimar Germany.

The Hungarian Mathematicians

Von Neumann was always particularly proud of the Hungarian mathematical tradition that produced him. Prior to the *Ausgleich*, Hungary's mathematicians were few, the best known of them being father and son, Parkas and Janos Bolyai.[14] The former studied at Göttingen, alongside Carl Friedrich Gauss, making original contributions in Euclidean geometry. The latter also worked on the problem of parallels, based on Euclid's Fifth Postulate, and was one of the independent creators,

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[13] See Tibor Frank, "Networking, Cohorting, Bonding: Michal Polanyi in Exile", *Polanyiana*, 2001, *10*:108-126.

[14] On the world of Hungarian mathematics, see Reuben Hersh and Vera John-Steiner "A Visit to Hungarian Mathematics", *The Mathematical Intelligencer*, 1993, *15*:13-26.

along with Gauss, of non-Euclidean, "hyperbolic" geometry. In the generation after Bolyai, several names stand out, both for their scientific work and their contribution to a national mathematical culture. Influential teacher and administrator, Gyula König completed his doctorate at Heidelberg in 1870, later joining the newly formed Technical University of Budapest in 1874, where he worked in algebra, number theory, geometry and set theory.[15] Also at the Technical University, József Kürschák worked in the fields of geometry, calculus of variations and linear algebra.

1894 was a pivotal year in the development of Hungarian mathematics, for this was the year in which Baron Eötvös Loránd (1848-1919), physicist, and founder of the Mathematical and Physical Society of Hungary, became Minister of Education. Like his politician father of the same name, Eötvös epitomized the Magyar liberalism of the late 19th century, under which the Hungarian Jews eagerly sought assimilation and became thoroughly attached to Hungary. Eötvös supported the establishment of both the Eötvös Competition in mathematics for secondary school students and the *Kozeposkolai Mathematicai Lapok*, or "*KöMaL*" for short, a monthly *Mathematics Journal for Secondary Schools*. The contribution to Hungarian mathematical culture of these two institutions, the competition and the magazine, is universally acknowledged.[16]

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[15] See the website in the history of mathematics at the University of St. Andrews, Scotland (www-history-mcs.st-andrews.ac.uk/history/Mathematicians.html). Each of the site's biographical entries is a synthesis of several sources, all cited and, in the present case, many of them in Hungarian.

[16] See, for example, Tibor Radó, "On Mathematical Life in Hungary", *American Mathematical Monthly*, 1932, 37:85-90 and Hersh and John-Steiner, "A Visit" (cit. n.14).

Promoted by Gyula König in particular, the prestigious Eötvös Competition was an annual examination intended to identify students of ability. Von Kármán (1881-1963) said that the toughest questions demanded true creativity and were intended to signal the potential for a mathematical career.[17] Over the years, in addition to von Kármán, the winners of the Eötvös Prize included Lipót Fejér, Gyula König's mathematician son Dénes, Alfred Haar, Edward Teller, Marcel Riesz, Gabor Szego, Laszl Redei and László Kalmár.

The mathematics magazine *KöMaL* was founded in 1894 by Győr schoolteacher Daniel Arány. Each issue contained general mathematical discussion, a set of problems of varying degrees of difficulty, and the readers' most creative or elegant solutions to the questions of the previous issue. Eagerly awaited in the postbox by many Hungarian students, it brought prestige to those who were successful, and contributed, like the Eötvös Prize, to the cultivation of a general interest in mathematics among the Hungarian young.

Schoolteachers of mathematics such as Arány, and also László Rátz and Mikhail Fekete of Budapest, played a important role in Hungary. As university positions were few, many mathematicians of fine ability found themselves teaching at secondary level or providing private tutorials to Budapest gymnasium students, the most talented of which they guided onwards towards

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[17] For a translation of a compilation of the problems, originally published in 1929, see József Kürschàk, *Hungarian problem book: Based on the Eötvös competitions, 1894-[1928]*, Rev. and edited by G. Hajós, G. Neukomm and J. Surányi, trans. by Elvira Rapaport (New York: Random House, 1963).

their university colleagues. In the case of von Neumann, his Lutheran Gymnasium teacher László Rátz was an important mentor, as were tutors Gabor Szegö and, later, Mikhail Fekete.[18]

In the development of a mathematical culture in Hungary, probably the most influential figure in the generation after König and Kürschák was Lipót Fejér.[19] Like his friend Max von Neumann, Fejér was born in Pécs. He distinguished himself in his contributions to *KöMaL*, won the Eötvös Prize in 1897, and studied mathematics and physics at the University of Budapest, spending a year at the University of Berlin. During this time, he changed his original name, Weiss, to the less Jewish-sounding Fejér, which also means "white" in Hungarian. Following a doctoral thesis at Budapest in 1902 on Fourier series, he taught in that city for three years, spending some time at Göttingen and Paris. After several years at Koloszvár, he won an appointment in 1911 to a chair in Budapest, where he would spend the rest of his career.

Budapest graph theorist Paul Turán would later credit Fejér with the entire creation of a coherent mathematical school. Another wrote that "a whole culture developed around this man. His lectures were considered the experience of a lifetime, but his influence outside the classroom was even more

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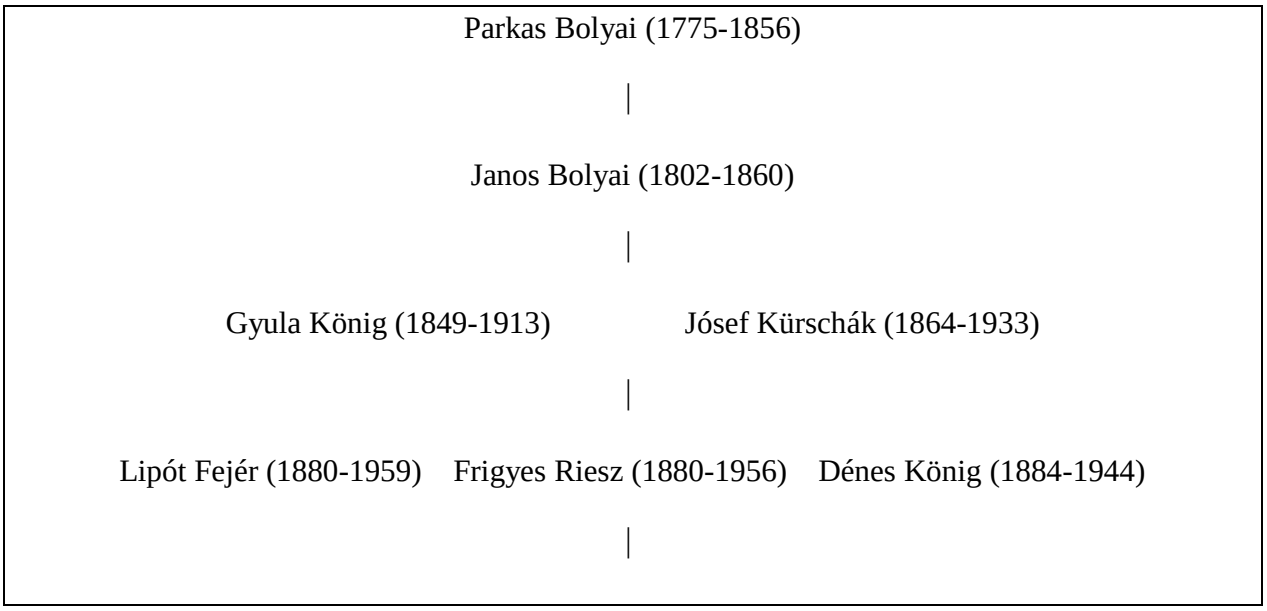
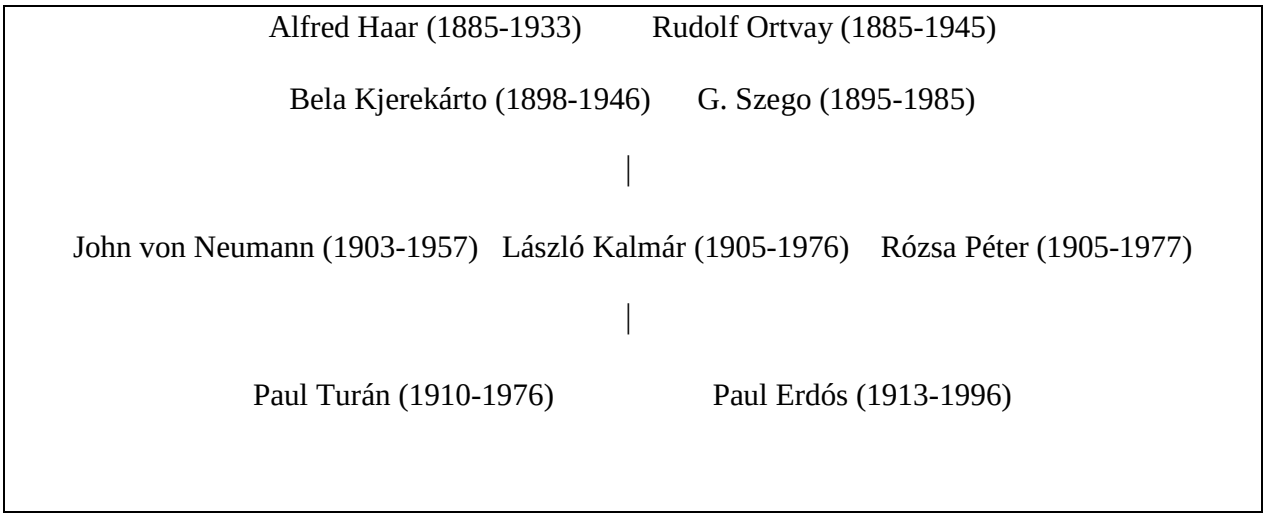
[18] See Lax, "Remembering John von Neumann" (cit. n.5) and also the discussion with Eugene Wigner in Aspray et al, "Discussion" (cit. n.5).

[19] On Fejér, see Paul Turán, "Fejér Lipót mathematikai munkásseaga", *Mat. Lapok, I*:160-170, translated as "Leopold Fejér's mathematical work", in *Collected Papers*, 1949, *I*:474-481, and Paul Turán, "Fejér Lipót, 1880-1959", *Mat. Lapok*, 1960, *12*:8-18, translated as "Leopold Fejér (1880-1959). His life and work", in *Collected Papers*, *2*:1204-1212.

significant".[20] Less formal in his contact with students, Fejér would sit in coffee-houses such as the Erzsébet café in Buda, or the Mignon in Pest, regaling his students with stories about mathematics and mathematicians he had known. A regular dinner guest at the von Neumann household, he apparently enjoyed the friendship of creative people of all sorts, including Endré Ady, the revered Hungarian poet. Beyond von Neumann, Fejér had a lasting influence on many younger Hungarian mathematicians, including George Pólya, Marcel Riesz, Gábor Szegô, László Kalmár, Rozsa Péter, Paul Erdós and Turán. Although the latter intimates that the events of 1919-1923, namely the Kun Revolution and the White Terror, left their mark on Fejér, until his death in 1959 he continued to enjoy an international reputation as one of the two recognized leaders of the Hungarian school of analysis. The other was his friend and close collaborator, Frigyes Riesz.



[20] G.L. Alexanderson, et al, "Obituary of George Pólya", *Bulletin of the London Mathematical Society*, 1987,*19*:559-608 quoted in Hersh and John-Steiner, "A Visit" (cit. n.14).



Riesz presided over the Franz Joséf University at Szeged, a provincial garrison town of 120,000 in the south of the country. This university had originally been in Kolozsvár, Transylvania, but had had to move when that region was handed over to Rumania in 1921. Riesz studied at Zurich Polytechnic, Budapest and Göttingen, before completing a doctorate in the Hungarian capital.[21] In 1911, he inherited Fejér's post at Koloszvár. Amongst the contributions for which Riesz is remembered is the famous Riesz-Fisher theorem, a central result on abstract Hilbert space that was essential to proving the equivalence between Schrödinger's wave mechanics and Heisenberg's matrix mechanics.[22]

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[21] See Frigyes Riesz, "Obituary", *Acta Scientiarum Mathematicarum Szeged, 7*:1-3, Edgar R. Lorch, "Szeged in 1934", (edited by Reuben Hersh) *American Mathematical Monthly*, 1993, *100*:219-230.

[22] Riesz's brother, Marcel, was also a mathematician of repute. Part of the Hungarian diaspora of the period, he made his career in Stockholm, Sweden.

Also in the Szeged group were Alfred Haar and mathematical physicist Rudolf Ortvay; of the latter, in particular, we shall have more to say later. In the mid-Twenties, they were joined by topologist Bela Kjerekárto, and two new assistants, István Lipka and Laszló Kálmár. Together, the Szeged mathematicians formed the János Bolyai Mathematical Institute, and they established the *Acta Scientiarum Mathematicarum Szeged*, or *Acta Szeged* for short, which published articles in the international languages and quickly became a mathematics journal of international reputation.[23]

Bolyai Institute of Mathematics, Szeged, 1928.

[23] See Bolyai Institute, "A Short History of the Bolyai Institute" (no date), available at server.math.u-szeged.hu/general/bolyhist.htm.

Back: Frigyes Riesz, Béla Kerékjártó, Alfréd Haar, Dénes König, Rudolf Ortvay

Middle: Józef Kürschák, Garrett Birkhoff, O. D. Kellogg (both visiting from U.S.), Lipót Fejér

Front: Tibor Rádo, István Lipka, László Kalmár, Pál Szász

(from Nagy 1987)

Dénes König, son of mathematician Gyula König, studied at Budapest and Göttingen, obtaining his doctorate in 1907, then becoming a teacher at his father's institution, the Budapest Technische Hochschule. His work represented an important stream in the Hungarian tradition, that of discrete mathematics, which includes graph theory, combinatorics and number theory. König lectured on graph theory and published a foundational book on it in 1936.[24] Amongst the younger mathematicians, closer in age to von Neumann, were László Kalmár and Rozsa Péter. The former was born in Kaposvár to the south of Lake Balaton and studied under Kürschák and Fejér, specializing in the field of logic. After a stay at Göttingen, he took a position at Szeged, initially serving as assistant to both Haar and Riesz. As for Péter, she was one of the very few women mathematicians of the period. Born Rózsa Politzer, she began studying chemistry at Loránd Eötvös University in Budapest, but switched to mathematics after attending lectures by Fejér.[25] Like Kalmár, to whom she was close, she graduated in 1927, specializing in number theory, but, being a

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[24] On König, see Tibor Gallai, "Dénes König: A Biographical Sketch", in Dénes König, *Theorie der endlichen und unendlichen Graphen.* (Leipzig,1936), trans. by Richard McCoart as *Theory of Finite and Infinite Graphs* (Boston: Birkhäuser, 1986), pp. 423-426. In the area of discrete mathematics, König's successors in the next generation were Paul Turán and Paul Erdós.

[25] On Péter, see Edie Morris and Leon Harkleroad, "Rózsa Péter: Recursive Function Theory's Founding Mother", *The Mathematical Intelligencer*, 1990, *12*:59-64.

Jew and a woman, she was particularly handicapped in obtaining a post as secondary school teacher. Depressed by the discovery that some of her theorems had already been proved by foreign mathematicians, Politzer actually abandoned mathematics, concentrating her energies on poetry and translation. It was Kalmár who encouraged her to return to the fold at the beginning of the 1930's, pointing to Gödel's recent results on incompleteness, which Politzer was then apparently able to reach using different methods. This led her to explore, in their own right, the recursive functions that had served as an important tool in Gödel's work, and she began presenting results in 1932, publishing several papers and eventually joining the editorial board of the *Journal of Symbolic Logic* in 1937. Notwithstanding a name change from Politzer to Péter, she remained without a post for a long time, making a living as a private tutor.

In this small community, von Neumann was quickly recognized as a prodigy. As a Gymnasium student, he caught the attention of Rátz, received tutoring in university-level mathematics from Fekete, and then enrolled at the University of Budapest, where he worked on set theory under the guidance of Fejér. Although registered there, he worked largely *in absentia*, part of the Hungarian student exodus, taking a parallel degree in chemical engineering at Zurich and then studying mathematics in Berlin. In 1926, when he went to Göttingen as International Education Board post-doctoral fellow under David Hilbert, he was already well-known to the German mathematicians.

Part II: Chess, Psychology and Mathematics

Von Neumann came to Göttingen to contribute to Hilbert's Formalist programme in the foundations of mathematics, and he soon became involved in the axiomatic treatment of quantum mechanics. It was for his work in these areas above all, especially the latter, which culminated in the publication of his 1932 *Grundlagen der Quantenmechanik*, that he achieved his reputation as brilliant young

mathematician.[26] His paper on games was presented early during his Göttingen sojourn, in December 1926. While congruent with Hilbert's desire to use the axiomatic method to clarify a range of fields, it also had its own genealogy, rooted in the rich world of chess.

In the early 20th century, the Royal Game was important throughout much of Europe, particularly in the countries of the Austro-Hungarian Empire. In Jewish culture, it was particularly prominent. From London to Moscow, the grandmasters enjoyed great prestige, and the game was played in the chess cafés of the capitals, such as Paris's famous Café de la Régence. Against a background of high tournament drama, chessmasters wrote manuals on strategy; psychologists investigated the thought processes required in the game; and mathematicians wondered whether so human an activity could be made amenable to formal treatment. Others speculated about the relationship of chess to life in general, and the game was source of inspiration for several writers, including Vladimir Nabokov, author of *The Defence* in 1929, and Viennese exile Stefan Zweig, whose *Schachnovelle* was the last thing he wrote before his suicide in Brazil in 1942.[27]

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[26] On Hilbert, see David E. Rowe, "Perspective on Hilbert", *Perspectives on Science*, 1997, *5*: 533-570; Leo Corry *David Hilbert and the Axiomatization of Physics (1898-1919)* (Dordrecht: Kluwer, 2004). On von Neumann and physics, see Miklós Rédei "Why John von Neumann did not Like the Hilbert Space Formalism of Quantum Mechanics (and What he Liked Instead)", *Studies in the History and Philosophy of Modern Physics, 27*: 493-510; Miklós Rédei and Michael Stöltzner (eds.*) John von Neumann and the Foundations of Quantum Physics* (Dordrecht: Kluwer, 2001).

[27] Stefan Zweig, *The Royal Game and Other Stories* (New York: Harmony Books 1981) orig. *Schachnovelle*, written in late 1941, early 1942, trans. as *The Royal Game* (New York: Viking

Looming large over this period is the figure of Emanuel Lasker (1868-1941), world chess champion for an unprecedented 24 years from 1897 to 1921. Trained as a mathematician, his mentors included Hilbert and Max Noether, and he completed a PhD in mathematics at Erlangen in 1902 on the theory of vector spaces. Having interrupted his studies to play chess for money, Lasker progressed rapidly to take the world title. Known and admired by Albert Einstein, Lasker was regarded as the player who introduced psychological considerations into chess. In this regard, he stood in contrast to previous world champion, Wilhelm Steinitz, and German champion, Siegbert Tarrasch, both of whom advocated a highly logical approach to the game, and the idea that, for every position, there existed a theoretically optimal move, independent of one's opponent's character.[28] This opposition between the logical and psychological approaches runs like a red thread through chess discussions of this period. Lasker often sought to deliberately destabilize his opponent by playing in a manner that did not correspond to any of the "textbook" sequences so thoroughly explored by Steinitz and Tarrasch. Was he blind, his opponents wondered, or did he have something hidden up his sleeve?

Press, 1944). For a fuller discussion of chess culture in relation to game theory, see Robert Leonard, *Von Neumann, Morgenstern and the Creation of Game Theory, 1900 – 1960* (Cambridge and New York: Cambridge University Press, forthcoming).

[28] On Lasker, see Jacques Hannak, *Emanuel Lasker: The Life of a Chess Master* (New York: Simon and Schuster, 1959), an English translation by Heinrich Fraenkel of the 1942 biography in German. More recently, see Ulrich Sieg and Michael Dreyer (eds.), *Emanuel Lasker: Schach, Philosophie und Wissenschaft* (Berlin: Philo, 2001).

Lasker was also a prolific author and his chess writings were unique in their richness, exploring not only strategy *per se* but also the connections between the game and other facets of life. If there exists an embryonic attempt to develop a "science of struggle" of relevance to the social realm, it lies in his 1907 pamphlet, *Kampf*, the short chapters of which bear titles such as "Strategy", "The Work Principle", "The Economy Principle" and "Equilibrium and Dominance". Here, Lasker uses his experience in chess as a point of departure to analyse the place of struggle in various realms, and economic ideas are the thread binding it all together.[29] Throughout *Kampf*, Lasker makes many references to the economic realm and to value, and gives central place to the figure of *homo economicus*.

Emanuel Lasker

It was to discussions of this kind that Lasker's contemporary, mathematician Ernst Zermelo was referring when, in a 1913 paper, he said he wanted to consider the game "in a mathematically objective manner, without having to make reference to more subjective-psychological notions such

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[29] Emanuel Lasker, *Kampf* (New York: Lasker's Publishing Co., 1907), reprinted in 2001 by Berlin-Brandenburg: Potsdam, with foreword by Lothar Schmid.

as the 'perfect player' and similar ideas". A keen chessplayer himself, Zermelo knew Lasker. Both were students in mathematics and they shared Hilbert as teacher.[30] At Göttingen, where Zermelo taught from 1897 till 1910, before moving to Zurich, there was considerable interest in chess. That university was also home to the oldest surviving handwritten document on the game, the *Göttingen Manuscript*, a Latin treatise on chess problems and openings, written by Portugese player, Lucena, in the late 15th century.

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[30] Ernst Zermelo, "Über eine Anwendung der Mengenlehre auf die Theorie des Schachspiels", *Proc. Fifth Congress Mathematicians*, Cambridge 1912, (Cambridge University Press, 1913), pp. 501-504, translated as "On an Application of Set Theory to the Theory of the Game of Chess" in Ulrich Schwalbe and Paul Walker "Zermelo and the Early History of Game Theory", *Games and Economic Behavior*, 2001, *34*:123-137, on p.? Zermelo (1871-1953) studied mathematics, physics and philosophy at Berlin, Halle and Freiburg, and his teachers included Frobenius, Max Planck, Lothar Schmidt and Edmund Husserl. His dissertation was completed at the University of Berlin in 1894. After two years as Planck's assistant, Zermelo went to Göttingen, where he was appointed *Dozent* in 1899. Beginning in 1902, he started to publish on set theory, and, in 1908, produced an axiomatics of set theory, which, improved by Fraenkel in the early 1920's, would become a widely accepted system. During the war, illness forced him to resign his chair at Zurich and leave academia. In 1926, after teaching private classes for a decade, he was given an honorary position at Freiburg. His 1935 refusal to give the Hitler salute would provoke a controversy at that university, causing him to withdraw from all teaching activity. On Zermelo, see Sanford L. Segal, *Mathematicians under the Nazis* (Princeton University Press, 2003), pp. 467-469.

Zermelo's treatment of chess was a purely formal set-theoretic characterization of the game. For example, he asks, if a player is in a winning position, is it possible to determine the number of moves necessary to ensure the win? His answer employs a proof by contradiction, showing that the number of moves in which a player in a winning position is able to force a win can never exceed the number of positions in the game. Were White, say, able to win in a number of moves greater than the number of positions, then at least one of the "winning positions" would have had to appear twice, in which case White could have adopted his winning moves when the winning position appeared the first time round, rather than wait till the second. Zermelo's paper clearly had nothing to offer the chess player, being of purely mathematical interest, yet there was enough in it to spark the interest of the Hungarian mathematicians in the mid-1920's, after World War I.

By this time, the psychological dimensions of chess were commanding greater interest than ever before. Already before the turn of the century, in response to the remarkable blind performances of American Richard Morphy, French psychologist Alfred Binet (1857-1911) had conducted a study of the psychology of blind chess play. In 1925, in an attempt to construct a profile of the gifted player of normal chess, a group of psychologists in Moscow performed *Gestalt*-type experiments on a group of chessmasters then participating in a tournament that city. One of the factors motivating that Russian study was the desire, in a Soviet state then turning against psychoanalysis, to repudiate the Freudian interpretation of chess then gaining currency. This reading, best captured, perhaps, in a famous 1931 essay by Freud's disciple Ernest Jones, saw chess as the expression of Oedipal desires, with its obsessive focus on immobilizing the King (father) and protecting the Queen (mother). Still in a psychological vein, 1925 also saw the appearance of a Russian silent film, *Shakmatnaya goryachka*, or "Chess Fever", in which the mental stability of the protagonist is threatened when he tries to play both black and white at the same time. In Nabokov's above-

mentioned chess novel, *The Defence*, the protagonist, Luzhin, when prevented from playing the game, begins to see every social encounter as a move or counter-move in a larger game. His descent into madness follows.[31] In short, the relationship between chess and the individual psyche was a richly speculative field, of interest to many in the interwar period. By concentrating on the formalism, the mathematicians, beginning with Zermelo, deliberately sought to circumvent such psychological questions.

Among von Neumann's Hungarian teachers, there was a long-standing interest in the mathematics of games. In 1905, in the columns of the *KöMaL*, a short paper by one Józef Weisz, "On the Determination of Game Differences", dealt with a game that was not one of pure chance. Throughout the 1920's, *KöMäL* founder, Daniel Arány, published papers examining how the probabilities of winning in games of pure chance varied with the number of players.[32] The Eötvös

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[31] See Binet, *Psychologie des Grands Calculateurs et Joeurs d'Échecs* (Paris, Genève: Slatkin, 1894); Djakow, Petrowski and Rudik, *Psychologie des Schachspiels* (Berlin and Leipzig: Walter de Gruyter & Co., 1927). On the psychoanalytical interpretation, see Ernest Jones, "The Problem of Paul Morphy: A Contribution to the Psychology of Chess", *International Journal of Psycho-Analysis*, January 1931, reprinted in Ernest Jones, *Essays in Applied Psychoanalysis*, Vol. 1, (New York: International Universities Press, 1964), pp. 165-196.

[32] See Józef Weisz, "Játékkülömbözetek Meghatározásáról", *KöMaL*, April 1905, pp. 185-6; Daniel Arány, "Note sur "Le troisième problème de jeu", *Acta Scientiarum Mathematicarum* (Acta Universitatis Szegediensis), 1924, *2*:pp. 39-42;"Verallgemeinerung des problems der Spieldauer für de fall von drei Spielern", *Mathematikai és Physikai Lapok*, 1927, *34*: pp. 96-105 (in Hungarian).

Competition of 1926, conceived by Dénes König, contained a question concerning the solution to a system of two equations, the answer to which was equivalent to proving that, on an infinite chessboard, in a sequence of appropriate moves, any square can be reached by a knight.[33] The intersection between games and mathematics was thus familiar territory to the Hungarians, so it is easy to understand why König and the younger Kalmár and von Neumann took a special interest in Zermelo's work as they passed through Göttingen in the 1920's.[34] There, the shadow of Lasker and the international excitement surrounding the game would have combined to make the subject particularly topical.

Reviewer: D. König, Budapest; "Sur la Généralisation du Problème de la Durée du Jeu pour Trois Joueurs", *International Congress of Mathematicians*, Bologna, 1928, pp. 73-75; "Considerations sur le problème de la durée du jeu", *Tohoku Mathematical Journal*, 1929, *30* :pp. 157-181; "Note sur le "Seconde problème de la durée de jeu dans le cas de trois joueurs"", *Association francaise Avancement Science,* 1929, *53*:pp. 33-35;"Le problème des parcours", *Tohoku Mathematical Journal*, 1933, *37*:pp. 17-22;"Le problème des parcours", *Assocation française pour l'Avancement des Sciences,* 1933, pp. 20-23. I thank Laszló Filep for drawing the Arány and Weisz papers to my attention.

[33] See Kürschák, *Hungarian Problem* (cit n. 17), pp. 104-106.

[34] Von Neumann was a chess player. We know, for example, that while in Zurich, in 1925-1926, he and his friend Willy Fellner, about whom we shall have more to say below, were both members of the Zurich Chess Club. I am grateful to Mr. Richard Forster, of Zurich, for this information.

Crediting discussions with von Neumann, each of König and Kalmár sought to refine Zermelo's 1913 paper. For example, in his 1927 "On a Method of Conclusion from the Finite to the Infinite", König applied to chess a lemma from set theory, at von Neumann's suggestion, in order to prove that the number of moves within which a player in a winning position can force a win is finite. To do this, König invokes the use of an infinite board, but with the usual 32 pieces.[35]

Standing apart from the above, another important contributor to the mathematics of games, was the French mathematician, Émile Borel. Unlike the Göttingen mathematicians, he was not interested in chess, taking Henri Poincaré's view that, since it could be played only on a board of 8 x 8 = 64 squares, with no possibility of generalizing to a board of n^2 squares, chess could not be considered a true mathematical object. In a series of notes written during the 1920's, Borel established the formal concept of strategy and investigated the range of 2-person, zero-sum games in which deliberately random play could be employed profitably. In various places, Borel considers applications, noting, like Lasker, that the "problems of probability and analysis that one might raise concerning the art of war or of economic and financial speculation, are not without analogy to the problems concerning games". Similar ideas are expressed in his writings of the 1930's, where he

[35] In his 1928/29 "On the Theory of Abstract Games", Kalmár generalizes the work of Zermelo and König, shows that if it is possible in a game to force a win, it can be done without the recurrence of any position.

suggests that the study of economic phenomena might be improved by incorporating the study of psychology and of games of chance.[36]

From Struggle to Equilibrium

"[O]ther mathematicians prove things they know, Neumann proves what he wants to prove"

Rózsa Péter[37]

In his pathbreaking paper, the 23–year old von Neumann goes beyond the work of Zermelo, the Hungarians and Borel, to lay out a theory of the generic strategic game, citing chess, baccarat,

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[36] The three most important of Borel's five published notes are: "La théorie du jeu et les équations intégrales à noyau symétrique", *Comptes Rendus de l'Académie des Sciences*, 1921 (December 19), *173*:1304-1308, translated in Maurice Fréchet, "Emile Borel, Initiator of the Theory of Psychological Games and its Application", *Econometrica*, 1953, *21*: 95-127; "Sur les jeux où l'hasard se combine avec l'habileté des joueurs", *Comptes Rendus de l'Académie des Sciences*, 1924, *178 :* 24-25; and "Sur le système de formes linéaires à déterminant symétrique gauche et la théorie générale du jeu", in "Algèbre et calcul des probabilités", *Comptes Rendus de l'Académie des Sciences*, 1927, *184 :* 52-53. Quotation taken from Borel, "La théorie du jeu", p.10. See also his "A propos d'un traité des probabilités", *Revue philosophique*, 1924, 98: 321-326 ; and *Traité du calcul des probabilités et de ses applications* (Paris: Gauthier-Villars, 1938), especially pp. X-XI.

[37] Rózsa Péter, *Játék a Végtelennel*. 1945, trans. Z. P. Dienes as *Playing with Infinity: mathematical explorations and excursions* (New York: Dover Publications, 1976, [1961c]) on p. 246.

roulette and poker as examples. He writes: "n players S_1, S_2, . . . S_n are playing a given game of strategy, G. How must one of the participants, S_m, play in order to achieve a most advantageous result?". The problem, he says, is well known, and "there is hardly a situation in daily life into which this problem does not enter". “[A]ny event”, he continues, “ - given the external conditions and the participants in the situation (provided the latter are acting of their own free will) - may be regarded as a game of strategy if one looks at the effect it has on the participants. What elements do all these things have in common?".[38]

The paper then proceeds to lay out, in complete precision, the definition of a game, along with the related concepts of strategy, payoff and probabilistic play. The bulk of the treatment is taken up with the 2-person game, where, implicitly answering Borel’s question, von Neumann demonstrates that such a game always has a solution, regardless of the number of strategies available to each player, i.e., each player can always choose his strategies, quite possibly drawing them in accordance with certain probabilities, in such a way as to keep his opponent’s expected payoff to a

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[38] Von Neumann, “Zur Theorie der Gesellschaftsspiele”, *Mathematische Annalen*, 1928, *100*: 295-320; translated by S. Bargmann as “On the Theory of Games of Strategy” , in *Contributions to the theory of games,* Vol. 4, ed. Albert Tucker and R. Duncan Luce. (Princeton, N.J.: Princeton Univ. Press, 1959), pp. 13-42, on p. 13. Here, in a footnote, von Neumann writes that this is the main problem of "classical economics: how is the absolutely selfish ‘homo economicus’ going to act under given external circumstances?" (p.13, fn. 2).

minimum (in this case: zero).[39] Although von Neumann makes no reference to Lasker, equilibrium in the 2-person game is characterized precisely by the disappearance of "struggle". Zermelo's resistance to psychologizing, too, is echoed by von Neumann, the existence of an equilibrium showing that "it makes no difference which of the two players is the better psychologist, the game is so insensitive that the result is always the same". Later in the paper, he promises a publication that will contain numerical examples of such two-person games as Baccarat, the game earlier treated by Borel, and a simplified Poker, which, by numerically demonstrating the necessity to "bluff" could be regarded as an "empirical corroboration of the results of our theory".[40] There was nothing mysterious about bluffing: it was simply rational play.

The paper closes with preliminary considerations on the 3-person, zero-sum game, and, specifically, the possibility that two players may do better by forming a coalition against the remaining one. As the rules of the game have nothing to say about which coalitions will actually be formed: "struggle" makes it reappearance here as the players compete for partners. Von Neumann concludes with the suggestion that a similar approach could be taken for games of 4 and, ultimately, any number of players, the result of which would be a "satisfactory general theory".

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[39] Von Neumann's proof involves a long and difficult argument, based on the lower- and upper-semi continuity of the functions bounding the two elements of the saddlepoint. For a detailed discussion of the mathematics, see Tinne Hoff Kjeldsen, "John von Neumann's Conception of the Minimax Theorem: A Journey Through Different Mathematical Contexts", *Archive of the History of Exact Sciences* 2001, *56*: 39 – 68.

[40] Von Neumann, "Zur Theorie der Gesellschaftspiele" (cit. n.38), p. 23 and p. 42.

However, he stopped there, and, as it turned out, did nothing further with game theory for over a decade.

Part III: From Games to the Social Order

To the New World

Notwithstanding the fact that his family had nominally converted to Christianity in 1923 upon the death of the father, in the climate of the late 1920's, von Neumann knew that his chances of obtaining a chair in mathematics in Germany were negligible. Stan Ulam remembered him speaking too of the worsening political situation, which made him doubt that intellectual life could be pursued comfortably. Also, many, many Dozents were competing for promotion. Thus, when, at the beginning of the 1930's, Princeton mathematician and occasional visitor to Göttingen, Oswald Veblen, arranged to have von Neumann spend six months per year at Princeton, the latter readily accepted. For the next two years, he commuted from Berlin to Princeton by cruise-liner, first-class as always, to a professorship in the Mathematics Department, which he shared with his compatriot Eugene Wigner.

Princeton's strength in mathematics in the 1930's resulted from its having two centres of gravity: the university's Department of Mathematics and the nearby, but independent, Institute for Advanced Study. The latter had been officially incorporated in 1930 through a large endowment by supermarket millionaires, Louis Bamberger and his sister Mrs. Caroline Bamberger Fuld. Also involved in the initiative was the Institute's first director, Abraham Flexner. It was decided to locate the Institute at Princeton University because of the its excellent library and the quality of its mathematics department, in whose building, Fine Hall, the Institute was first located. The first full faculty member, secured by Flexner in 1932, was Albert Einstein, then keen to leave Germany and

being courted by universities the world over. By the time the Institute opened its doors in the fall of 1932, Veblen, von Neumann and James Alexander were on the faculty, having transferred from Princeton's mathematics department. Einstein physically arrived in 1933. The Institute, which at the outset had only a School of Mathematics, paid lavish salaries. In 1933, it moved to new, purpose-built quarters, just south of Princeton campus, where it has remained to this day.[41]

Princeton Mathematics was known for its strength in topology and algebra, two relatively young fields, important in the growth of American mathematics. Veblen was a leader in combinatorial topology, a field in which Alexander and Solomon Lefschetz also worked. Other mathematicians included Bob Robertson, who would become a close friend of von Neumann, Luther Eisenhart, Alonzo Church, Marston Morse, Carl Siegel, Albert Tucker and statistician Sam Wilks. In addition to the quality of faculty, the social occupation of space seems to have been important, for all those in reminiscence about Princeton mathematics point to the importance of the Fine Hall Common Room, where afternoon tea and the playing of games made for a certain *esprit de corps*, quite unlike the mathematics departments at Columbia or Harvard, where nothing comparable existed.

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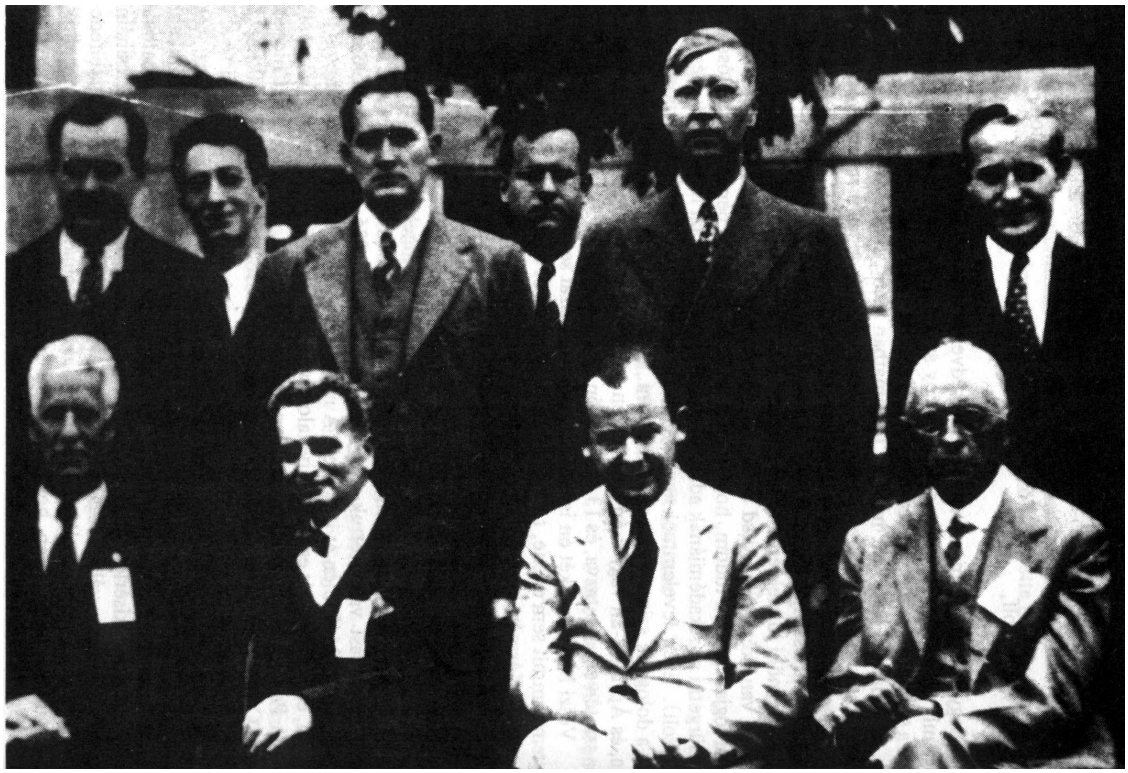
[41] On the Institute see Steve Batterson, *Pursuit of Genius: Flexner, Einstein and the early faculty at the Institute for Advanced Study* (Wellesley: A. K. Peters, 2006); Ed Regis, *Who got Einstein's office?: Eccentricity and Genius at the Institute for Advanced Study*, (New York: Addison Wesley, 1988).



Von Neumann and mathematicians at Princeton

(from Nagy 1987)

From the beginning, von Neumann took greatly to America. Although he would always dress formally – including when on horseback and on the beach – he seems to have appreciated the freshness of life in the States. Arriving with his first wife, Mariette Kovèsi, herself from a prominent family in Budapest, where her father was head of the Jewish Hospital, they soon moved into a large house in Princeton with domestic staff. There, they threw famous parties, complete with caviar imported from Russia. The Depression was kind to them. Virtually every summer, though, they returned to Hungary, fleeing the heat and humidity of the East Coast, just as his teachers continued to flee that of Budapest.

Into Disequilibrium

Von Neumann had no reason to regret his decision to leave Germany. In mid-March 1933, a few weeks after the Reichstag fire, and days after the sweeping Nazi election victory under Hitler, he wrote from Budapest to Flexner back at the Institute. He had hoped to spend the Summer lecturing in Berlin, but the "newer german developments" (sic) had thrown this into question, and he didn't think that conditions would improve.[42] A week or so later, in April, the German "Restoration of Civil Service Act" was passed, marking the beginning of the systematic dismissal of Jewish faculty members from the German universities. Flexner wrote from Princeton, condemning the German government's madness, and its destruction of the Göttingen faculty in particular.[43]

To Veblen, von Neumann wrote about American economic affairs in detail, and about European politics with irony: "There is not much happening here, excepted (sic) that people begin to be extremely proud in Hungary, about the ability of this country, to run into revolutions and counter-revolutions in a much smoother and more civilized way, than Germany. The news from Germany are bad: heaven knows what the summer term 1933 will look like. The next programme-number of Hitler will probably be the annihilation of the conservative-monarchistic - ("Deutsch National" = Hügenberg) – party". He continued:

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[42] John von Neumann to Abraham Flexner, March 18, 1933, Faculty Files, John von Neumann, Folder "1933-35", Von Neumann Papers, Institute for Advanced Study (hereafter VNIAS).

[43] Flexner to von Neumann, May 6, 1933, Faculty Files, John von Neumann, Folder "1933-1935", VNIAS. On Göttingen after 1933, see Segal, *Mathematicians* (cit. n.30).

"You have probably heard that Courant, Born, Bernstein have lost their chairs, and J. Franck gave it up voluntarily. From a letter from Courant I learned 6 weeks ago (which is a very long time-interval now in Germany), that Weyl had a nervous break-down in January, went to Berlin to a sanatorium, but that he will lecture in Summer.

I did not hear anything about changes or expulsions in Berlin, but it seems that the "purification" of universities has only reached till now – Frankfurt, Göttingen, Marbürg, Jena, Halle, Kiel, Königsburg – and the other 20 will certainly follow.

I am glad to learn from your letter that these things received the full attention and appretiation [sic] in America which they deserve. It is really a shame that something like that could happen in the 20th century".[44]

He chose not to go back to Berlin. After a leisurely summer, with weekends spent on Lake Balaton and in the Hungarian countryside, he returned permanently to the Institute at Princeton, never setting foot in Germany again.[45]

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[44] VN (Budapest) to Veblen, April 30, 1933, Veblen Papers, Library of Congress (hereafter VLC), Box 15, Folder 1.

[45] William Aspray, in *John von Neumann and the Origins of Modern Computing* (Cambridge, Mass.: MIT Press, 1990), indicates that von Neumann visited Germany, Austria, Hungary, Italy, and France in 1930 and 1931; Germany and Hungary in 1932; Hungary and Italy in 1933; Hungary alone in 1934; England and Hungary in 1935; and France in 1936; and Hungary alone in 1937 and 1938 (p. 256, n. 35).

Amongst his Hungarian correspondents, an important mentor was the above-mentioned Rudolf Ortvay, a physicist eighteen years his senior. Born in 1885 in Miskolc, Ortvay too studied at Göttingen and, following a period at Koloszvár and Szeged, moved to Budapest in 1928, where he ran the Institute for Theoretical Physics. Like Lipót Fejér, he was a family friend of the von Neumann's. He followed the young mathematician's career from the beginning, and they corresponded throughout the 1930's.[46]

[46] The von Neumann-Ortvay letters, written in Hungarian, are located in the Library of the Hungarian Academy of Sciences in Budapest and in the von Neumann papers at the Library of Congress, with copies in the Stan Ulam papers at the American Philosophical Society in Philadelphia. There are 60 of them, running from May 9, 1928 to February 16, 1941. Most of them have been reproduced in Hungarian in Ferenc Nagy, *Neumann János és a "Magyar Titok", A Dokumentumok Tükrében* (Budapest: Országos Müszaki Információs Központ és Könyvtár, 1987), and a few in English in Miklos Redéi (ed.), *John von Neumann*: *Selected Letters* (Providence, RI: American Mathematical Society: 2005). All the quotations in this paper are based on translations of the letters found in Nagy, *op cit*, conducted for the author by Mr. Michael Szirti.

Rudolf Ortvay in the 1930's

(from Nagy 1987)

By late 1933, Ortvay had become quite pessimistic about European politics, and his resonant letters from here on were at once a lament for cultural decline, a inquiry into the place of the scientist in society, and a meditation upon the vagaries of the human spirit. Von Neumann's replies, in turn, not only shed light on his character, but gradually reveal the extent to which his social scientific reflections and political preoccupations were intermeshed. From 1934 onward, while continuing his mathematical work on the spectral theory of Hilbert space, ergodic theory, rings of operators and Haar measure, he was increasingly preoccupied by politics, entering into the finest detail in his letters. Faced with a relatively emotional Ortvay, he tended to maintain a certain detachment: "What you write about the uncertainty of the future of European civilization is regrettably plausible.

There is one consolation in it, but even this isn't an excessively certain conclusion: the war demoralised principally the countries that lost, and in history after a lost war experimentation with a state structure of tyranny or dictatorship, and the rise of a romantic, irrational nationalism, is neither a new nor rare phenomenon. Naturally references to historical analysis are especially arid and hopeless, since if these could be trusted, then new wars could not be avoided".[47]

In November 1934, von Neumann wrote, wondering whether the two German Rockefeller fellows arriving at the Institute were Nazis. "How do you now judge Central Europe?" he continued, where "the situation ... seems to be so tense that in the end there will be trouble! There are so many uncertain and easily misunderstood circumstances in the European 'balance' that there may exist a government that jumps into an adventure".[48] England and Italy were equally indecisive, or, rather, hypocritical, he said, and the weight of Russia was just as incalculable as it had been in 1914. Later that month, he had grown sombre: "The European political situation appears to be quite dark even from here…; to wit, here the people have already accepted that the lesson was for nought, and that in Europe there shall be a war in the next decade".[49]

Von Neumann's rupture with the German mathematical community was made final early the following year, when he responded to the infamous affair involving Ludwig Biberbach, the *Deutsche Mathematiker Verein* and Danish mathematician Harald Bohr. In an attempt to

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[47] Von Neumann to Ortvay, January 26, 1934.

[48] Von Neumann to Ortvay, November 2, 1934.

[49] Von Neumann to Ortvay, November 28, 1934.

demonstrate the relevance of mathematics to the new regime, and to justify the ouster of many Jewish mathematicians, prominent German mathematician and Nazi sympathiser, Bieberbach, had taken a novel position in the long-standing philosophical debate between the Intuitionists and the Formalists. Intuitionist mathematics, he claimed, was more truly German, insofar as it emphasized the concrete and the empirically relevant. Formalist mathematics, on the other hand, was too given to purely abstract manipulations and tended to be favoured by Jewish mathematicians. Bieberbach went very far in this matter, seeking to connect different mathematical styles to the race psychology of Erich Jaensch, and establishing *Deutsche Mathematik*, a new journal that would promote a truly German mathematics. When, in May 1934, Harald Bohr published an article condemning Bieberbach, the latter responded with an open letter excoriating Bohr and his "hatred of the new Germany".[50] At the meeting of the DMV in Bad Pyrmont in September of that year, Bieberbach's followers succeeded in having a resolution passed expressing regret for his behaviour but also condemning Bohr's. The affair met with great consternation amongst mathematicians internationally, and it drew a polite but firm letter from von Neumann to William Blaschke of the DMV: "Although not a German, . . . I had received my scientific education in the German speaking part of the World and have spent part of my scientific career in German Universities – a part, which remains for me unforgettable for ever… Nevertheless I cannot reconcile it with my conscience to remain a member of the German Mathematical Society any longer… It is my hope that my paths

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[50] Quoted p. 221 in Herbert Mehrtens, "Ludwig Bieberbach and "Deutsche Mathematik", in Esther Phillips (ed.), *Studies in the History of Mathematics*, (Washington D.C.: The Mathematical Assocation of America, 1987), pp. 195-241. On the Bieberbach affair, see also Segal, German Mathematicians (cit. n.30), pp. 263-288 and Ch. 7, passim.

and those of the D.M.V., whose true interests I still believe to be serving, are not separating for ever".[51]

It was at this time that Polish mathematician, Stan Ulam, entered von Neumann's life. Ulam would later remember their first meeting on the train platform in Warsaw in 1934, when von Neumann was returning from a Moscow conference with Birkhoff and Marshall Stone: "The first thing that struck me about him were his eyes – brown, large, vivacious, and full of expression. His head was impressively large. He had a sort of waddling walk… At once I found him congenial. His habit of intermingling funny remarks, jokes, and paradoxical anecdotes or observations of people into his conversation, made him far from remote or forbidding".[52] The two hit it off immediately, both being what Ulam describes as third or fourth-generation wealthy Jews, comfortable with each other, and linked through mutual acquaintances, Ulam's widowed aunt having married Árpád Plesch, one of the richest men in Budapest. When Ulam moved to the States in 1936, he was von Neumann's assistant at the Institute, before being appointed Junior Fellow at Harvard. They were to become closest friends, their solidarity no doubt helped by what they would traverse together.

Throughout the mid-1930's, von Neumann devoted his main efforts to continuous geometry, carrying on this work even when on holiday in the Canadian woods with the Flexners, but his concern for politics deepened by the month. By January 1936, he was writing to Ortvay about the

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[51] Von Neumann to W.J.E. Blaschke, January 28, 1935, in Rédéi (ed.) (2005) John von Neumann (cit. n. 46). Original in German; translation by M. Rédei.

[52] Ulam, *Adventures* (cit. n.12), p. 67.

effect of Mussolini's Italy on the European situation, and continuing his dire predictions: "Here Europe is judged darkly, as with every affair that is distant and complicated. But even I cannot bring myself to tranquility. The danger of war appears to be truly great, even if the catastrophe does not take place this year. I hope that from near by, the picture is not this desolate. How do you judge it?".[53]

In late 1936 or early 1937, von Neumann gave a popular talk at Princeton on what, according to the *Science News Letter*, was for him "a mere recreation", his analysis of games and gambling. All of it appears to have referred to the work he had done at Göttingen a decade previously. There was no mention of anything other than 2-person parlour games. He spoke about "stone-paper-scissors", showing that by "making each play the same number of times, but at random, ... your opponent will lose in the long run".[54] Also briefly reported are his comments on the probabilities of making particular plays in both dice and a simplified poker.

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[53] Von Neumann to Ortvay, April 1, 1936.

[54] *Science Letter News*, April 3, 1937, "Princeton Scientist Analyzes Gambling: "You Can't Win", p. 216. Merrill Flood, Princeton graduate student, and later a mathematician at the RAND Coporation, remembered attending the talk: "He lectured on the minimax theorem, although he didn't call it that… He gave us examples of how mixed strategies could be used in games. It made a great impression on me, and I remember going to Kleene and Einstein and half a dozen other people to find out if they had ever heard of that. . . Nobody came up with the idea of mixed strategy among all these bright people. That convinced me that that's a subtle thing". Interview with Merrill Flood by Albert Tucker, San Francisco, May 14, 1984, Transcript No. 11 (PMC11) of

Then, towards Christmas 1937, von Neumann's personal life became complicated, when Mariette left him for Desmond Horner Kuper, Princeton graduate student in physics and a regular guest at their parties. Although Ulam would later say that the rupture greatly shook his friend, in his correspondence at least, von Neumann bore it all with equanimity: "Many thanks for your letter . . . and particularly for what it contained about my 'domestic' complications. I am really sorry that things went this way - but at least I am not particularly responsible for it. I hope that your optimism is well founded - but since happiness is an eminently empyrical (sic) proposition, the only thing I can to is to wait and see . . .".[55] Emotional detachment notwithstanding, events took their toll on him. As his marital difficulties became intermeshed with developments in Hungary, he entered a critical period in which his normally volcanic output of papers collapsed: to one in 1938, and less the year after.

A Time of Instability

The main reason why von Neumann returned to Hungary in mid-1938 was so that he could marry his second wife-to-be, Klára "Klari" Dán, and bring her back to the U.S. Though part of the same privileged Budapest circle, they had only met properly a few years previously. It was in Monte Carlo, where she was holidaying with her first husband, Francis:

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oral history project *The Princeton Mathematics Community in the 1930's*, deposited in the Seeley Mudd Library, Princeton University.

[55] Von Neumann to Ulam, Oct. 4, 1937.

> "When we walked into the Casino, the first person we saw was Johnny; he was seated at one of the more modestly priced roulette tables with a large piece of paper and a not-too-large mound of chips before him. He had a "system" and was delighted to explain it to us: this "system" was, of course, not foolproof, but it did involve a lengthy and complicated probability calculation which even made allowance for the wheel not being "true" (which means in simple terms that it might be rigged). Johnny was a little bit bashful about his "system" and insisted that he really did not believe in it; nevertheless he was determined to test it thoroughly.
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> Francis went on to another table. For a while I wandered around watching the lunatic pleasure of people destroying themselves, then I went to the bar and sat down, wishing I had company with my drink. As I was sipping my cocktail, Johnny appeared. I shall never forget the meek and apologetic way he sidled up to by table and asked if he might join me. "Of course", I said, "pull up a chair; I hate to be a lonely drinker". Johnny, a little embarrassed, but with the cute cunning of a child who wants his ice cream but will not ask for it directly, exclaimed: "A drink – what a splendid idea – I would love to have one with you, but are you sure that you can afford it? You see, you will have to pay for mine. My system did not quite work and I am completely cleaned out"[56]

Von Neumann's charm evidently worked, for by February 1938, with he newly divorced, she had left her second husband and begun sitting out the 6-month waiting period before her divorce proceedings could begin. It was the protracted nature of the separation that brought von Neumann even closer to European political developments.

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[56] Klari von Neumann-Eckhart "Johnny", pp. 10-11.

Early in March 1938, in a well-known speech at Győr, close to the Austrian border, the Hungarian prime minister, Kálmán Darányi, outlined his plans for concrete legal measures designed to cope with the “zsidókérdés”, the "Jewish Question". “I see the essence of the question in the fact that the Jews living within Hungary play a disproportionately large role in certain branches of the economic life, partly owing to their particular propensities and positions and partly owing to the indifference of the Hungarian race. Their position is also disproportionate in the sense that they live to an overwhelming extent in the cities, and above all in the capital. . . . The planned and legal solution of the question is the basic condition for the establishment of a just situation – a just situation that will either correct or eliminate the aforementioned social disproportions and will diminish Jewry’s influence. . . to its proper level”.[57] With the *Anschluss* of Austria a fortnight later, the Győr Programme became something of a national obsession in Hungary, giving rise to a 3-month parliamentary debate on Bill No. 616, designed to ensure the "more effective protection of the social and economic balance".

This was clearly a difficult time for von Neumann. His wife had just abandoned him, and the situation in Europe was looking increasingly ominous. Writing to Ortvay after the *Anschluss* of Austria, he admitted to being even more pessimistic than him. The catastrophe could not be avoided.[58] It was not a case of proving why it would happen, he said, but why it would not. He was certain that, if there were no other means to ensure an English victory, the U.S.A. would

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[57] Quoted in Braham, *The Politic*s (cit. n.10), on p. 121.

[58] Von Neumann to Ortvay, March 17, 1938.

intervene on England's behalf, the latter being essential to U.S. security in the Far East. He was also very interested in how domestic politics in Hungary would be affected by Austria's demise. Ortvay's pessimism deepened in turn. Even putting aside the danger of a catastrophic war, he said, he judged the whole development of culture very darkly. The "advance of the masses" was a negative feature of early 20th century modernity: the development of the popular press, the "adoration of the automobile and machinery", the excesses of propaganda, mass travel - this was "modern barbarianism, with all its technical superlatives as described so nicely by A. Huxley", and it prevented the emergence of a higher form of life. The problem was not how to further satisfy the masses, said Ortvay, it was, rather, how to keep them under control. The obvious need for a strong moral stance, in scientists given that it could not be expected in politicians, served to underline the importance of emotions and spiritual qualities. Yet never before, wrote Ortvay, had there been so great a gulf between the scientist's technical capacities and his level of culture or moral state. It caused him great anguish daily, he said.[59]

In April, von Neumann fled Princeton, returning to Hungary to be close to Klari. It was the beginning of a short *Wanderjahre*, spent hovering around Budapest and travelling around Europe - waiting until his domestic stability was restored, watching as his own country changed by the month.

In May, Hungarian Bill No. 616 became Law No. XV, the famous "Balance Law", the purpose of which was to reduce to 20% the proportion of Jews in the professions and in financial, commercial and industrial enterprises of 10 employees or more. Those to be exempted included war invalids

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[59] Ortvay to von Neumann, April 4, 1938.

and those who had converted before August 1919 and their descendants. The aims of the law were expected to be achieved within 5 years, through the dismissal of 1,500 Jewish professionals every six months. With the support of the Church and the liberal-conservative leaders of the gentry and old feudal order, the rules of Hungarian life were changing.

The law was also a response to the popular perception of injustice as regards Jewish privilege, strategically undertaken in such a way as to dampen the claims of the Hungarian radical Right, the *Nyilas*. Under their leader, Szálassi, the latter were clamoring not only for much harsher measures against the Jews, pointing to Germany, but also for significant reforms in the area of land ownership and the franchise. The *Nyilas* were thus feared as a genuine threat to the traditional semi-feudal order. Indeed, when Darányi appeared to be too close to the popular Right, he was ousted and replaced as Prime Minister by Béla Imrédy. Other attempts to stall the Far Right included Horthy's forbidding civil servants to join extremist political parties, in April 1938, and the Interior Minister's banning the *Nyilas* Party less than a year later. Observing this, von Neumann could write to Veblen: "I am familiarized by now with the state of mind, the bellyaches and the illusions of this part of the world – such as they are since the annexation of Austria. The last item (illusions) is rather rare, the preceding one not at all... Hungary was well under way of being Nazified by an internal process – which surprised me greatly – in March/April. The new government, which was formed in May stopped this process, or slowed it down, but for how long, is not at all clear".[60]

In June, he was in Warsaw, for a conference organised by the League of Nations' International Institute for Intellectual Cooperation, in which several physicists including Bohr and Heisenberg

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[60] Von Neumann to Veblen, June 8, 1938, VLC, Box 15, Folder 1 ?, emphasis in original.

took part. He also gave a talk to Ulam's former teachers and colleagues, including the logicians Knaster, Kuratowski and Tarski. Ulam later travelled down to Hungary to join von Neumann, visiting Budapest and travelling with him through the countryside. They visited von Neumann's teachers Fejér and Riesz at Lillafüred, near Miskolc, an attractive forested area in the mountains, and favourite resort of the Hungarian elite. Ulam would later recall their walking through the forests together, talking about the possibility of war. Afterwards, he returned northwards to Poland, by train through the Carpathian foothills: "The whole region on both sides of the Carpathian Mountains, which was part of Hungary, Czechoslovakia, and Poland, was the home of many Jews. Johnny used to say that all the famous Jewish scientists, artists, and writers who emigrated from Hungary around the time of the first World War came, either directly or indirectly, from these little Carpathian communities, moving up to Budapest as their material conditions improved". When later asked why these Jews were so creative, von Neumann felt that it was "a coincidence of some cultural factors which he could not make precise: an external pressure on the whole society of this part of Central Europe, a feeling of extreme insecurity in the individuals, and the necessity to produce the unusual or else face extinction".[61] In the Summer of 1938, when Ulam and von Neumann were there, the pressure on the area was real. For several months, Hitler had been dangling before Hungary the promise of the return of Subcarpathian Ruthenia and Slovakia, should the Hungarians cooperate with his plans for the rest of Czechoslovakia. Hungary held back, keen to involve Great Britain along with the 3rd Reich and Italy in settling these East Central European disputes.

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[61] See Ulam, *Adventures* (cit. n.12), p. 111 and p. 114.

By late Summer, Klari had sent von Neumann away from Budapest, claiming that his meddling in the matter of her divorce was only making things worse. The tension surrounding the matter, exacerbated, it appears, by what Klari found to be von Neumann's childish manner, left her sounding quite desperate at times. Her almost daily letters, many of which were written from the finest hotels and resorts in Lucerne, Venice and Montecatini, are pervaded by signs of depression and even hints at suicide. Von Neumann's letters, written in locations ranging from Lund to Abbazia, are filled with attempts to placate and reassure her, apologizing for earlier tantrums and promising that the future will be better. It would prove to be a durable epistolary pattern. But, soon, political anxieties began to dominate. By late August, Klari could write that it had been decided that her sister, Böske, and children should absolutely leave the country, in a matter of days: "I don't know what fate will bring us, things look very dangerous at present and maybe in a few days we shall have such worries we won't have time to think of this (sic) [divorce] questions anymore".[62]

Isolated in various hotel rooms, Klari seems to have had few friends in whom she could confide, and she soon began to write about wishing to see Vilmos "Willy" Fellner and his wife Valerie "Vally" Koralek.[63] Having written the previous day about Budapest being in a "frantic state", she

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[62] Klari (Grand Hotel & La Pace, Montecatini Terme) to vN, Aug. 28, 1938, VNLC, Box 1, Folder 7.

[63] Like the von Neumann's and the Dán's, the Fellner's were a prominent assimilated family, their fortune going back to the 1860's. Von Neumann and Fellner had attended the same Budapest *gymnasium* and were students together in chemical engineering at Zurich, where they shared the

spent the evening of September 18 with them at a film and a late-night circus cabaret. Klari and Willy Fellner talked politics till three in the morning: "[E]ven a huge snake fully alive", she wrote blackly, "could not disturb our happy projecting of who is now going to be killed. Well I suppose this is what happens if two full-blooded pessimists meet. Poor Vally again tried to persuade us to watch the show or at least not to use certain names too often as the place was terribly crowded and people seemed rather interested in our opinion". "I'm so worried that I don't talk of this (sic) matters with my family anymore. I don't want to know them (sic) how terribly scared I am. I don't know what's awaiting us in the future, but never as long I may live will I forget 1938... If you go to England I should very much like an objective report from you whether the Jewish question is really getting so bad there as I heard."[64]

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same address. Fellner would later say that it was von Neumann and another Hungarian friend named Imré Revesz, (later Emery Reves, and confidante of Churchill), who were responsible for sparking his interest in economics, in which he completed a PhD in Berlin in 1929. Returning to Budapest, he was involved in the family manufacturing business (sugar, alcohol and paper). Like a number of cultivated non-academics, Fellner pursued an active interest in economics, although without publishing anything of note during that period. See James Marshall, "Fellner, William J." in Sobel, R. and B.S. Katz eds., *Bibliographical Dictionary of the Council of Economic Advisors* (New York, 1988), Gottfried Haberler, "William Fellner In Memoriam", in W. Fellner, *Essays in Contemporary Economic Problems, Disinflation* (Washington and London, 1984), pp. 1-6 and Irma Adelman, "Fellner, William John", *Encyclopaedia of the Social Sciences* (?), p. 301.

[64] Quoted letter, Klari to vN, Sept. 11, 1938, VNLC, Box 1, Folder 7.

While, with Klari, von Neumann deliberately appeared optimistic, in his letters to Veblen he was less sanguine. From Lund, where he was visiting Marcel Riesz, he wrote: "I agree with you, that war at this moment is improbable, since neither side seems to want it just now – but the Sudeten-german (sic)- population seems to be very nearly out of control, so you can never tell. It also seems, as if Messrs. H[itler]. and M[ussolini]. were a little more emotional lately than rational, so you really cannot tell. So we may be much nearer liquidation than it seemed 2 weeks ago. God knows what will happen...".[65]

Back in Budapest in early October, von Neumann could write to Veblen that the Munich Non-Aggression Treaty between Chamberlain and Hitler had provided welcome breathing space. Following that agreement, Imrédy had visited an aggressive Hitler at Berchtesgaden, with no satisfactory conclusion as regards the Czech territories. A month later, however, Hungary's claims were submitted to German-Italian arbitration. This resulted in the First Vienna Award, made official in November, which granted the *Felvidék* in southern Czechoslovakia to Hungary. Imrédy sought re-election that month and formed a new government. All of this provided respite for von Neumann. With Klari's divorce secured, they married in late November, and, a fortnight later, sailed together for the U.S. The Fellner's had already left. Like the vast majority of Jews, however, von Neumann's and Klari's families clung to Hungary, soothed for the meantime by the Munich outcome.

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[65] VN (Grand Hotel, Lund) to Veblen, Sept. 15, 1938, VLC, Box 15, Folder 1 ?

Rationality and Pathology

From Princeton, the von Neumann's watched events unfold in Hungary. If the retrieval of the *Felvidék* had been welcomed by all Hungarians, for whom Trianon had been a injustice, it also brought with it a population of 1 million, including several orthdox Jewish centres. Thus emerged that contradictory feature of Hungarian politics during this period: territory was regained, satisfying a need shared by all Hungarians, but, with it, came pockets of orthodox Jews, the effect of which was to inflame anti-semitism. By December, Imrédy was promoting a second anti-Jewish bill "Concerning the Restriction of the Participation of the Jews in Public and Economic Life". Then, in a strange twist, he himself was unable to refute an accusation by the radical Right that there was Jewish blood in his own ancestry, which compelled him to resign in February 1939. Horthy swore in Pál Teleki a few days later. A renowned academic and cartographer, the aristocratic Teleki was tolerant of the Magyarized Jews, but less so of the "Ostjuden". This became more topical an issue with Hungary's acquisition of Subcarpathian Ruthenia in March 1939, which brought with it a substantial Jewish Orthodox population, whose urban politicised intellectuals were left-leaning.[66] This development stimulated the parliamentary debates on the second Jewish law, which took place in the first half of 1939. This second bill was more "Nazi" in content, referring not only to the threat to economy and culture but also to the racial, psychological and spiritual difference of the Jews. Anxiety grew in Budapest. On New Year's day, 1939, Ortvay could write to von Neumann that Leo Libermann, an opthalmologist and university professor known by both of them, had just

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[66] On the Jews of Subcarpathian Ruthenia, see Livia Rothkirchen, "Deep-Rooted Yet Alien: Some Aspects of the History of the Jews in Subcarpathian Ruthenia", *Yad Vashem Studies*, 1978, *12*: 147-191.

committed suicide. "In the state of the world, one cannot find great joy, I see it as slipping downward… ".[67]

If Ortvay persisted in searching for psychological depth, throughout this time, von Neumann resisted it. Yet, at the same time, von Neumann was ready to speak of what he called the "pathology" of the general situation. It was difficult to write about politics, he admitted, and especially difficult to be sure that his diagnosis was not simply the expression of his own desires - "*Wunschbestimmt*" – but he felt reasonably objective about the matter: the war was inevitable, he said, and the arguments that it was not necessary, or that it would not resolve the problems, were beside the point. "The whole affair", he wrote, "is a pathological process and, viewed clinically, is a plausible stage of further development. It is 'necessary' even emotionally - if it is permissible to use the word 'necessary' in this connection. It will bring the acute problems to a resolution insofar as it will diminish the moral and intellectual weight of the European continent and its vicinity, which, considering the world's structure, is justifed. May God grant that I am mistaken".[68]

In his letters, the emphasis shifted subtly from the inevitability of catastrophe to the question of what would follow it. Apologising to Ortvay for not delving into the mathematics of the "spirit", i.e., emotions and attitudes, he dwelt persistently on politics, with the vocabulary of structure and equilibrium creeping into his prose. Point by point, he went through the issues. It was naive to hope that any outcome would be useful to the Jewry stranded in Europe. One possibility was an

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[67] Ortvay to von Neumann, Jan. 1, 1939.

[68] Von Neumann to Ortvay, Jan. 26, 1939.

outcome similar to "the Turkish-Armenian affair during the World War" - the genocide of Armenians by the Turkish government - to which Hitler had referred in a recent speech, an outcome which, von Neumann said, was "superfluous to analyse".[69] Even if this did not occur, he said, in the vanquished countries there would be social chaos and lasting division between the various sides, making it impossible that a "state of equilibrium could take place". A victory to the Western powers, he said, would be in many respects Pyrrhic, with the rapprochement of the dissatisfied, in the form of a German-Russian coalition, posing a future threat at least as worrying as the present one. Economically, the U.S. stood to gain little from a war. The wartime boom would be only temporary, with debts incurred never being repaid and the American social structure dangerously loosened. Speculating on the possibility of American imperialism emerging in the event of their victory in war, von Neumann felt that this would be possible "only if the war liquidates Japan too", which wasn't completely out of the question. The war, he agreed with Ortvay, would indeed be a terrible cultural loss in Europe - indeed, such a loss was already being incurred - but neither should one exaggerate: when the Romans took over Greek culture, the ancient civilization remained essentially intact for another 300 years. "After all this", he concluded, "I believe the war is plausible in spite of all, and with the relatively early participation of the U.S.A. Because it is a pathological procedure, which does not take place because anyone considered it intelligently, that it is in his interest, but because certain abnormal spiritual tensions - which no doubt exist today in the world - search for 'resolution' in this direction. And because from a rational point of view, England

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[69] The Armenian catastrophe of 1915 held an important place in the imagination of many German and Central European intellectuals during the interwar period, thanks largely to Franz Werfel's interwar novel inspired by the affair, *The Forty Days of Musa-Dagh*, trans. from German by Geoffrey Dunlop (New York: Viking Press, 1934, [1933c]).

and France cannot let one another perish, nor can the U.S.A. let England".[70] The pessimistic diagnosis, he said, on which they now seemed to agree, was now much closer to reality.

Replying at length, Ortvay felt that Western Europe was in decay, as evidenced by its excesses of capitalism and mechanisation, its shallow rationalism, "which consists in the fact that a few easily comprehensible viewpoints are fulfilled to the extreme", and its “excessive cult of the will”, which conferred power upon "a very aggressive, half-cultured mass". America, although hampered by the absence of an aristocracy, still showed signs of cultural health and force, and thus bore a responsibility for regeneration – indeed, for the future of humanity. If only a minority there, he implored von Neumann, could substitute for the absent aristocratic class, and set an example for the rest of the population. He realised how non-modern his thinking was: it was, he admitted, as if he were living in Herder’s time...

For over a year, at that point, emboldened by Hitler’s advances, Hungary’s German-speaking Swabians had been growing vocal in their demands for increased economic and cultural autonomy. By 1939, they had become an important political presence, providing a direct link with the 3rd Reich. As previously with Czechoslovakia, Germany wanted to have Hungary’s support for its designs on Poland. Hungary resisted, Poland being an old ally, but it was also keen to placate

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[70] Von Neumann to Ortvay, Feb. 26, 1939. In his letters, von Neumann makes several references to the history of antiquity, in which he was well-read. One of his favourite books was Thucydides’ *The Peloponnesian Wars*, in which he was particularly fond of the Melian dialogues, a model of rationalist, *realpolitik* discourse.

Germany, whose support it would need in its own claims on Transylvania, which it wanted to retrieve from Rumania.

If the law of 1938 had met with the surprise, but not opposition, of the Jewish population, the second anti-semitic bill brought protest. The Hungarian Jews proclaimed their patriotism, pointing to their sacrifices during the Great War, to their contribution to the economic, cultural and scientific life of the country. They turned to the British Jews for assistance. In February 1939, Szálassi's followers launched a grenade attack on people leaving Budapest's Dohàny St. Synagogue. In May, the second law was enacted, prohibiting Jews from obtaining citizenship (something aimed at recent refugees and those residents in the recently acquired territories), and ordering the retirement of all Jewish court and prosecution staff by 1940, and primary and secondary teachers by 1943. Reintroducing the 1920 *Numerus Clausus* of a 6% limit on admission to universities, it also prohibited Jews as editors or publishers of periodicals or producers or directors of plays or films. Licenses held by Jews for various kinds of businesses were to be withdrawn. Firms of 5 employees or less could have 1 Jew, while those of 9 employees or more could have 2.[71]

Jewish historian, Ralph Patai, has written about the devastating cumulative psychological effect of the laws of 1938 and 1939: "Even if the laws did not immediately endanger their lives . . . the new

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[71] On the economic impact of the Jewish laws on the Hungarian economy, see Kádár and Vági. Gábor Kádár and Zoltán Vági "Rationality or Irrationality? The Annihilation of Hungarian Jews", *The Hungarian Quarterly*, 2004, *XLV*: ?; Gábor Kádár and Zoltán Vági, *Self-Financing Genocide: The Gold Train, the Becher Case and the Wealth of Hungarian Jews* (Budapest & New York: Central European University Press, 2004).

situation demanded a total rethinking of their own position in Hungary, something of which most Hungarian Jews were simply incapable".[72] Patai goes on to describe how that attachment to Hungary left many of them somewhat paralysed. Many shunned Zionism, so that even though the 1939 law made express provision for the emigration of Jews from Hungary – subject, of course, to financial restrictions – relatively few resorted to it.

Von Neumann's family and his in-laws were among those reluctant to leave, so that, that summer, in July 1939, Klari returned to Budapest from Princeton to try to persuade them to do so. While she was gone, Ulam and von Neumann slipped away for a few days to visit Veblen at his Summer home in Maine. On the way, Ulam recalls, they "discussed some mathematics as usual, but mostly talked about what was going to happen in Europe. We were both nervous and worried; we examined all possible courses which a war could take, how it could start, when".[73] When Hitler overran Poland the following month, Ulam felt as if a curtain had fallen on his past life, cutting it off from his future. His father, sister and many other relatives were still in Poland. "This was the period of my life when I was perhaps in the worst state, mentally, nervously, and materially. My world had collapsed . . . There was a terrible anxiety about the fate of all those whom we had left behind – family and friends".[74] With Klari away in Europe for several weeks, von Neumann's anxiety, too, reached a new pitch: "What are your further plans in Europe? Your father should not hesitate any longer. . . Can he not make up his mind?. . . Don't be untimely (sic) sentimental, you

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[72] Patai, *The Jews of Hungary* (cit. n.10), p. 541.

[73] Ulam, *Adventures* (cit. n.12), p. 115.

[74] *Ibid*, p. 118.

might be the one who saves them by insisting on talking rationally!".[75] That August, after some delays, von Neumann's mother and brother arrived in New York. After further delays, the Dán family, too, left Budapest for the States. Ulam's family would not escape from Poland.

As if in response to the tension, Ortvay's letters now ran to several pages, ranging on subjects from axiomatics to God to Freud. He hoped that the European nations would wake up before European culture collapsed entirely. A desirable solution would see, not one side crushed by the other, but an entente, where each recognised the other's virtues, their right to exist, as well as their failings. With Freud's death in London that year, Ortvay spoke of his long-standing interest in Freudianism and of having been in contact with several of the psychoanalyst's followers, with sometimes unpleasant experiences. Freud, he said, had provided the first systematic exploration of the psychology of the subconscious and of repression. Ortvay was prepared to acknowledge the importance of sexuality, but not to the extent suggested by Freud. Drives such as aggression, the will to power, revenge and envy were important, as, in a few people, were higher spiritual emotions. The Freudian view was quite unbalanced, Ortvay felt, and its success lay in its being drilled into his followers. Through effective propaganda, it took on a political or religious dimension. Yet, looking at the war and the events leading to it, Ortvay felt that he could not deny the great importance of repression and of sharply distinguishing between the causes superficially believed to be important and the underlying mechanisms: "I believe that these are economic forces only to a very slight degree; rather they are enormously primitive and brutal passions, and the 'economic' reasons are in many cases only

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[75] Von Neumann to Klari, August 12, 1939 (year unmarked), KEMNW. His letters of this interlude, which also allude to Klari's father's depression, culminated in an August 24 telegram: «PLEASE TAKE FIRST AVAILABLE SHIP PLEASE WIRE LOVE».

suitable for the purpose of letting modern man hide the real reasons from himself... Nietzsche already saw a great deal here".[76]

These discussions of rationality and pathology, of politics and society, saw von Neumann return to the mathematics of games. In November, he was planning to spend part of the following Summer at the University of Washington, Seattle, at the invitation of Abraham Taub. In a letter to the department suggesting possible topics for some additional popular lectures, he included the theory of games: "I wrote a paper on this subject in the Mathematische Annalen 1928, and I have a lot of unpublished material on poker in particular. These lectures would give a general idea of the problem of defining a rational way of playing. I think that even stating the problem is not at all trivial and leads to a number of quite amazing considerations on the nature of games like chess on the one hand, and of another kind on the other hand, of which – I think – poker is the prototype... The discussion of games played by more than two persons leads to further questions which can also

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[76] I have been unable to determine whether or not Ortvay had contact with Sandor Ferenczi, Freud's principal interpreter in Hungary. According to von Neumann's brother, Nicholas, Ferenczi was a close relative of the von Neumann family and psychoanalysis a frequent topic of family conversation. See Nicholas A. Vonneuman, *John von Neumann as seen by his brother* (Meadowbrook, PA: N.A. Vonneuman, 1987) on p. 36. On Ferenzci's life see Arnold W. Rachman, *Sandor Ferenczi: The psychotherapist of tenderness and passion* (New York: Jason Aronson, 1997).

be discussed in a manner which I think will interest the intelligent but non-technical audience".[77] A week later, he returned to Ortvay: "Unproductive as it is to meditate upon political problems, it is hard to resist doing so. Maybe from Hungary the meaning of the European, and particularly East-European, situation's elements are clearer. But from here it makes a fairly complicated and confused impression. In particular, it appears in all likelihood that not 2, but 3 or 4, enemies are facing one another".[78] The European situation, one might say, was not a 2-person game.

A week before Christmas, 1939, the psychological difficulties of forced emigration were brought home to the von Neumann household in the starkest manner, when reluctant exile Klari's father, Károly "Charles" Dán, committed suicide at a train station near Princeton. The Weyl's and others rallied round the von Neumann's in their difficulty. From the Institute, Veblen's secretary wrote to him with the news:

> "Mrs. von Neumann came to call on me yesterday afternoon! I hope she did not feel under any kind of compulsion… But it seemed to some satisfaction to her to talk. She looked

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[77] Von Neumann to Prof. Carpenter, Nov. 29, 1939, VNLC, Container 4, File 3, Personal Correspondence 1939-40.

[78] Von Neumann to Ortvay, Dec. 8, 1939. In the same letter, he worries about the Polish mathematicians, many of whom, at this time, were being shot by the Nazis or sent to labour camps. For the names of the dozens of murdered mathematicians, see Kazimierz (Casimir) Kuratowski, "A Half Century of Polish Mathematics", *Fundamenta Mathematicae*, 1945, *XXXIII*: v-ix. See also *Annals of the Polish Mathematical Society*, 1945, *XVIII*: i-iv.

> shrunken, but did become natural in talking of general conditions – in England now for instance. She said she has now no courage to try to dissuade her mother from returning as soon as possible to Hungary; that she had insisted on her parents' coming here as the only best course she could then see. Now she questions whether alternative courses might not have been better. I told her it seems to me we must in such cases rest on the assurance that we did what seemed best at the time (which we should probably do again in the same conditions, with the same experience). Professor Weyl also has been conscious of this special cause of her depression.
>
> … She also would like Professor von Neumann to get at least a few days rest away from Princeton, "even 20 miles away". But she herself apparently needs it as much as anyone. She says she has been closely confined by her father all fall, conscious of his abnormality, trying to help him, and not wanting to expose his condition to other people".[79]

This event would prompt von Neumann to take his wife with him on his visit to the West coast. In March, he confirmed with Seattle that he would give three evening lectures on games, covering "The case of chess; The notion of the "best strategy"; Problems in games of three or more players". He would leave in May and drive across the country.[80]

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[79] December 27, 1939, Mrs Blake to Veblen, VLC, Box 15, Folder 1

[80] Von Neumann to Prof. Carpenter, March 29, 1940, Container 4, File 3, Personal Correspondence 1939-40, VNLC.

Ortvay continued to write about mass movements and war, insisting that rational, utilitarian, considerations played only a secondary role: the fundamental reasons were "primitive passions". This conformed to the Freudian mode of thinking, he said, but the passions were different from Freud's. Anything which challenged our self-worth evoked hate, which, in the case of mass movements, was directed towards destroying the object of the animosity. Even in business, where utilitarian considerations were perhaps strongest, a fundamental force was often the suppression of a competitor, who simply could not be tolerated, and not just for reasons of profit. Passions of this kind, he felt, were at the root of the last war, the present one, and the antisemitic movements as well.[81]

It was around this time that Oskar Morgenstern became a significant presence in von Neumann's life. Though not Jewish, Morgenstern had begun to consider leaving Vienna in the mid-1930's, when the situation was becoming increasingly uncomfortable. When, in 1938, he finally chose the then sleepy Princeton University, part of his motivation was the presence of the Institute nearby. This was how he got to know von Neumann and his colleagues. He first met the Neumann couple in early 1940 at the home of Hermann Weyl, at what appears to have been one of their first outings after their recent bereavement. By March, von Neumann had become "Johnny" to Morgenstern and he was showing an interest in the economist's concerns for the problems of foresight and decision. They drew closer in April, when the relationship between Morgenstern's concerns in economics and von Neumann's work on games began to become clear. Before heading out West, von Neumann read and praised some of Morgenstern's earlier work. Under the mathematician's

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[81]Ortvay to von Neumann, March 30, 1940. In this connection, Ortvay asked for von Neumann's help for their friend, Rozsá Péter, who had lost her position because of the Jewish law.

bracing influence, the economist turned to Richard von Mises' (1939) *Kleines Lehrbuch des Positivismus,* and regretted not having abandoned sooner the universalism of Othmar Spann and the idealistic philosophy he had encountered in Vienna.

The Trip out West

By mid-May, von Neumann and Klari were driving across the U.S., on the way to Seattle, where he was to lecture from mid-June to end-July. Once again, von Neumann was on the move, this time into the heart of America, driving westwards, their backs turned to Europe, as if to flee the source of their troubles. From a hotel in Winslow, Arizona, he wrote a long, rich letter to Ortvay: "The travel is quite dreary until the middle of Kansas, but from then on the land is incredibly beautiful and varied – I am really ashamed that for 10 years I have always put it off till "next year". Furthermore, the most beautiful parts, the Grand Canyon, Northern California and Oregon are still ahead of us". But there was no getting away from politics: "Naturally, from the perspective of bringing Europeans over here, all that can be said is that the bottom has fallen out of the world – I don't even dare to think what the disintegration of the Scandinavian countries, the Netherlands, Belgium (and tomorrow and the day after, who knows what else?). But even if these – and other evident political possibilities – make even the slightest degree of success doubtful, I will do everything I can".

Turning to science, Neumann agreed with Ortvay that theories that were unduly complicated could not be right. For these reasons, he was especially "horrified" by biochemistry. "I cannot accept", he said, "that a theory of prime importance, which describes processes which everybody believes to be elementary, can be right if it is too complicated, i.e, if it describes these elementary processes as being horribly complex and sophisticated ones". He could not substantiate this with any detailed

knowledge, but he felt it intuitively. There was a need for new terminology and new models in several fields.

One area where von Neumann sought simplicity was in “politics and psychology”. Thus, here, although he agreed with much of what Ortvay had written, he could not go with him entirely. We quote von Neumann at length:

> "I too believe that the psychological variable described by you, where resentment is the primary attitude, and the “egotistic-”, “profit motive” only a secondary and (often not even quite plausible) rationalization – is an oft-occurring and important psychological mechanism. But neither is it permissible to forget entirely the other variable either: selfishness, in a wrapping of principles and ethics... In the present conflict, particularly given the antecedents, I would still find it difficult to believe that the enemies of the Germans are moved by mainly by the first mechanism.
>
> Concerning the practical chances, and the future, and what would be desirable... It is difficult to write about this, since the letter will travel for 3-4 weeks, and this time interval is not “negligible”. You know that I do not believe 'compromise' to be either desirable or possible. The survival of the German power in any form signifies, among other things, the rapid liquidation of the European "Vielstaatlerei" [federation]. I don't believe that this would be a factor ensuring equilibrium from a small European nation's point of view. If the allies are victorious, then without doubt they will orient Europe to the "Vielstaatlerei". Viewed from afar, this is a retroactive development, but from the viewpoint of small European nations e.g. the Hungarian nation, it is the only chance at all. To speak of a German counterweight against Russia, I believe, is an impractical daydream.

> That the war, in the case of the Western Allies also, even if they are victorious, will result in the extension of state power and the impoverishment of today's economically leading classes is very plausible to me too. But I believe that this has to be interpreted as follows:
>
> If, in physics, it can be shown of a procedure that it is accelerated by all disturbances and entirely independent of the disturbance's nature, and clearly accelerated more the greater the disturbance - then it is usual to assume that the procedure leads to a state of equilibrium. This is most likely true in politics as well. Further, in politics, even more complicated is the fact that if such a procedure is carried out by means of a given political movement, then it soon becomes clear that efforts directed towards combatting this movement serve as at least a good mechanism in the same direction...
>
> I don't believe that cultural wealth would be less in a centralised society than in the old, free economy. Although such a thesis could be defended dialectically, its opposite, I believe, could be defended just as well. Empirically, all that is clear is that the transition is harmful, but this, naturally, is no miracle.
>
> Returning to the purely political theme: I don't see how both sides could acknowledge the other's raison d'être: If the German nation's frame of mind, which evolved during the last ten years, does not end with a very obvious cataclysm, then no one else on this earth has a raison d'être".[82]

Did all of this – this insistence on simplicity, whether in scientific theories or in human motivation; this likening of political to physical equilibrium - speak to the "*Wunschbestimmt*" that von Neumann had written of previously? Were they projections of his own desires, signs of his hopes

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[82] Von Neumann to Ortvay, May 13, 1940.

for order, beyond the inevitable cataclysm, in the same spirit as his earlier reminders that Greek civilisation had remained intact long after the Roman conquest?

Klari would later remember this journey out West as being filled with drama. It was May 1940, and the Allies were suffering one setback after another. Rather than spend two days as intended in Denver, the couple stayed a week, to avail of newspaper extra editions and continuous broadcasts: "Holland was being invaded the day after we arrived in Denver. . . By the time the negotiations for the surrender of Belgium had started, we had made it to Nevada".[83] She remembered von Neumann spending hours beside the car radio, or requesting that the continuous chain of news be switched on even during social gatherings. "Then, as soon as the news was being told, Johnny would start a running comment of his own, giving his interpretations of the day's events".[84] Continuing northwards from Pasadena, where they presumably saw von Karman, to the Fellner's at Berkeley, they heard the news of the fall of France.[85] In, Metford, Oregon, their car broken down, they spent the night in a tiny hotel, where the radio "was blasting President Roosevelt's famous 'stab in the back' speech. The Italians were coming in on the German side and all the little Central European nations were jumping on the Nazi bandwagon": "We must have played at least a hundred games of Chinese checkers that night (of course, Johnny won all of them), but all throughout he kept talking, going over and over the same arguments, like the broken record running in the same

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[83] Klari "Johnny" (cit. n.2), p. 21.

[84] Ibid, p.24.

[85] Fellner had managed to secure a lecturing position in Economics at Berkeley when he and his wife fled Hungary in 1938.

groove; he was repeating the details of the last weeks' tragic events and then proving that, in spite of all the adversities, the Germans were going to lose in the end. He talked with the obsession of a maniac who, however, had clear logical arguments to prove his case".[86]

Mathematics and Social Stability

That journey marked a significant turning point in von Neumann's life. Once back at Princeton, he plunged into war work, beginning as consultant on mathematical statistics and aerodynamics to the Ballistics Research Laboratory of the Army Ordnance Department at the Aberdeen Proving Ground in Maryland.[87] It was the beginning of a complete immersion, marked by the same peripatetic frenzy that had previously carried him around Europe or across the States. In 1941 and 1942 he would become a member of Division 8 of the National Defense Research Council (NDRC), working mainly on high explosives, in particular the shaping of charges. In 1942 and 1943, he would work for the Section for Mine Warfare of the U.S. Navy's Bureau of Ordnance, Research and Development Division, on operations research, spending the last part of 1942 in Washington. The work brought him ceaselessly up and down the Eastern seaboard, and from January to July 1943 he was away in England. Later that year, he began as consultant to the Manhattan Project at Los Alamos, in time bringing Ulam in with him.

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[86] Klari, "Johnny", pp. 25-26.

[87] In 1940, the War Department formed a Scientific Advisory Committee, including von Neumann and physicists I. I. Rabi and Theodor von Karman, to periodically review the functioning of the Laboratory. See Aspray (1990), p. 26ff.

Ahead of all that, however, upon returning to Princeton in August 1940, he plunged into game theory.[88] One characteristic of his working practice as a mathematician was his apparent need for an interlocutor, even a passive one, in certain phases. That summer, the person was Israel Halperin, his only ex-doctoral student, who would go to the house on Westcott Rd., where von Neumann would: "go over ideas or create them, and fill my head full of this stuff for an hour and a half. Then he would tell me to come back the next morning. . . It was my impression that he wasn't just talking about it, he was doing the work, and that the reason he sent me home after each morning was that he wanted to think alone for a while. . . I realized I was right at the beginning of something very hot, but it wasn't the sort of thing I felt comfortable with".[89] Morgenstern was, obviously, a more active interlocutor. At this point, he was independently pursuing ideas that had grown out of an earlier paper on the difficulties of assuming perfect foresight in economic theory.[90]

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[88] Discussions with Fellner at Berkeley were clearly important, for no sooner was von Neumann back on the East Coast than he wrote to him, clearly in the light of earlier conversations. The letter thanks him for reminding von Neumann of a paper by Gerhard Tintner, which he was reinterpreting in the light of game theory, and shows that von Neumann had begun working out a concept of "solution" to the 3-person game. Von Neumann to Willi Fellner, Aug 15, 1940, von Neumann Papers, National Technical Information Centre and Library, Budapest, original and translation kindly provided by Mr. Ferenc Nagy.

[89] Halperin Interview with Albert Tucker, May 25, 1984, Princeton University, Princeton Mathematics Community in the 1930's, Transcript Number 18 (PMC18).

[90] Morgenstern's thinking is best displayed in an unpublished 1940 draft, "Maxims of Behavior", which drew on Karl Menger's book *Morale, Wille und Weltgestaltung*, (Vienna: Julius Springer,

Morgenstern and Von Neumann, Sea Girt, New Jersey

(courtesy of Mrs. Dorothy Morgenstern-Thomas)

Nonetheless, von Neumann worked away independently, going beyond his theorem of a decade previously, exploring games of three and more players, creating the concepts of coalitional equilibrium and stability, searching for a full theory. By October, he had produced an unpublished

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1934), in an attempt to tackle the circular difficulties of modelling interaction between economic agents. See Leonard, "Ethics and the Excluded Middle: Karl Menger and Social Science in Interwar Vienna", *Isis*, 1998, *89*: 1-26.

typed draft "Theory of Games I (General Foundations)".[91] Following a presentation of the 2-person, zero-sum case, he turns to presenting the set function v(S) for the *n*-person game. It shows the value (i.e., gains) available to a coalition of players, S, which, by complete internal cooperation, plays minimax against its complement. He conjectures that this set function, v(S), will be sufficient to determine the strategies to be adopted for the entire game by each of the *n* players. "We now study the special case n = 3 for a clue as to what we should mean by a solution to our problem. Assuming a fully normalised game, v(S) is here uniquely determined by … :

	0		0
	-1		1
v(S) =	1	for a(S) =	2
	0		3

Clearly then the advantageous strategy is for any two players to form a coalition against the third: by this the set will gain, and the third lose, one unit".[92]

Von Neumann describes how the apportionments between the three players are determined by the above set function. Each member of the "winning coalition" will receive 1/2. Were either of them

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[91] Von Neumann, "Theory of Games I (General Foundations)", OMDU, File John von Neumann, 1940-1948.

[92] *Ibid*, p. 12. The column on the right shows the number of players in the coalition; that on the left each coalition's winnings.

to insist on more, the other could profitably deflect to form a coalition with the "defeated" player. Also, no player can improve his chances of entering a winning coalition by offering to accept less than 1/2, for the other two players would compete with each other to join him, thereby eroding away the premium offered. "So we see: each of the two members of the "winning" coalition gets 1/2 . . . and the formation of any particular one among the three possible "winning" coalitions cannot be brought about by paying "compensations" and the like. Which "winning" coalition is actually formed, will be due to causes entirely outside the limits of our present discussion".[93] These external causes were sociological or other features, not reflected in the rules of the game, that restricted or promoted the formation of particular coalitions. Here lay the limits of the theory. It carried the analysis up to the point where such social influences entered the picture and it showed how they mattered, but could say little about where they came from.

Von Neumann begins his search for a general definition of stability with the 3-person, zero-sum game above, where there are three possible outcomes, each comprising a coalition of two winners against a single loser.

> "(4.b) None of them "can be considered a solution by itself – it is the system of all three and their relationship to each other, which really constitute a solution.
>
> (4.c) The three apportionments possess together, in particular, a certain "stability" to which we have referred so far only very sketchily. It consists in this, that any strategic course, followed by a majority of the players, will ultimately lead to one of them. Or, that no equilibrium can be found outside of these three apportionments.

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[93] Von Neumann, "Theory of Games I", p.13.

(4.d) Again it is conspicuous that this "stability" is only a characteristic of all three apportionments together. Neither one possesses it alone – each one, taken by itself, could be circumvented if a different coalition pattern should spread to the necessary majority of the players.

We will now proceed to search for an exact formulation of the heuristic principles which (sic) lead us to our solution…

A more precise statement of the intuitive "stability" of the above system of three apportionments may be made in this form: If we had any other possible apportionment, then some group of players would be able and willing to exchange it for one of the three already offered, but within the system of given apportionments we cannot find a group of players who find it both desirable and possible to exchange one scheme for another…"[94]

To extend the solution concept to the case of the general *n*-person game, he develops further notation and terminology. A coalition is effective for a particular valuation (later called an imputation) if, by forming a coalition, members may find it possible to get as much as the valuation offers them. Thus, it becomes possible to speak of a valuation, α, *dominating* another, β, if there exists a non-empty set, S, effective for α, for which $\alpha_i > \beta_i$ for all members of S. For the *n*-person game, the solution can be defined as a collection of valuations, ν, such that:

(i) for every $\alpha, \beta \in \nu$, it is never the case that $\alpha > \beta$ (i.e., no imputation in the solution is dominated by any other member imputation) and

(ii) for every $\alpha' \notin \nu$ there exists an $\alpha \in \nu$ for which $\alpha > \alpha'$ (i.e., every imputation outside the solution is dominated by at least one imputation inside).

He proceeds to discuss the properties, in a manner quite different from that done earlier with the 3-person game. He notes that the definition of a solution has not ruled out the existence of a α' where $\alpha' > \alpha$, i.e., the existence of imputation lying outside the solution which dominates at least one of the member-imputations, and therefore would be preferred by some effective coalition. His defence of the definition of solution in the face of such a possibility is most interesting:

> "If the solution ν, i.e., the system of valuations, is "accepted" by the players 1, . . . n, then it must impress upon their minds the idea that only the valuations $\beta \in \nu$ are "sound" ways of apportionment. An $\alpha \notin \nu$ with $\alpha' > \beta$ will, although preferable to β, fail to attract them, because it is "unsound". [For the 3-person game, he refers here to the earlier explanation of why a player will be averse to accepting more that 1/2 in a coalition]. The view on the "unsoundness" of α' may also be supported by the existence of an $\alpha \in \nu$ with $\alpha > \alpha'$. [i.e., the mere presence in the solution of a third imputation that dominates the "dominating" non-member, α', may be sufficient to deter players from seeking α']. All of these arguments are, of course, circular in a sense, and again dependent on the selection of ν as "standard of behavior", i.e., as a criterion of soundness. But this sort of circularity is not unfamiliar in everyday considerations dealing with "soundness".
>
> If the players have accepted ν as a "standard of behavior", then it is necessary, in order to maintain their faith in ν, to be able to discredit with the help of ν any valuation not in ν. Indeed for every outside α' ($\notin \nu$) there must exist an $\alpha \in \nu$ with $\alpha > \alpha'$.

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[94] Ibid, p. 14.

> . . . The above considerations make it even more clear that only ν in its entirety is a solution and possesses any kind of stability – but none of its elements individually. The circular character stressed [above] makes it also plausible that several solutions ν may exist for the same game – i.e., several stable "standards of behavior" in the same factual situation. Each one of these would, of course, be stable and consistent in itself, but conflict with all others".[95]

Von Neumann then devotes several pages to a graphical illustration of the solutions to the 3-person, zero-sum, normalized game, which he uses to illustrate the distinction between proper and improper solutions, the first being a solution set that is finite, the latter being one that is infinite.

> "The example 7.B also indicates one of the major reasons which lead to improper solutions. There, one player – it happens to be 2 – is being discriminated against, for no intrinsic reason, i.e., for no reason suggested by the rules of the game itself, which are perfectly symmetrical. Yet a "stable standard of behavior", i.e., a solution ν can be built up on such a principle. This player has a – rather arbitrary – value assigned to him: $\alpha_2 = b_0$ for all valuations $(\alpha_1, \alpha_2, \alpha_3) \in \nu$. He is excluded from the competitory (sic) part of the game, which takes place between the other players exclusively -1 and 3.
>
> This discrimination, however, need not be clearly disadvantageous to the player who is affected. It is disadvantageous if $b_0 = -1$. But we can also choose $b_0 > -1$, as long as $b_0 < 1/2$. At any rate, however, it amounts to an arbitrary segregation of one of the players from

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[95] Ibid, pp. 17-18.

> the general competitive negotiations for coalitions, an arbitrary assignment of a fixed – uncompetitive - value for this player in all valuations of the solution, and all this causes an indefiniteness of apportionment between the other players".

Von Neumann closes by noting that subsequent discussions will show that there may be other causes of improper solutions, all of which "can be interpreted as expressing some arbitrary restriction on the competitive negotiations for coalitions which does nevertheless permit the definition of a "stable standard of behavior"".[96]

Thus did von Neumann work out a theory of equilibrium coalition formation in games of various sizes and degrees of complexity. Morgenstern was a crucial catalyst and interlocutor, plunging into the work at night and weekends when von Neumann could free himself from the war work, writing an introduction to the *Theory of Games and Economic Behaviour* - the only part read by many.[97] The mathematical architecture, however, was all von Neumann's, and the stable set was the central solution concept, with the bulk of the book devoted to its exploration. While that exploration is enormously ramified and complex, given the combinatorial complexity of certain games, the importance of social norms in determining equilibrium outcomes remains fundamental throughout.

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[96] Ibid, pp. 26-27.

[97] On the collaboration with von Neumann, see from Morgenstern's point of view, see Leonard, «From Parlor Games to Social Science: von Neumann, Morgenstern and the Creation of Game Theory, 1925-1944 », *Journal of Economic Literature*, 1995, *XXXIII*: 730-761.

As von Neumann worked out his new theory, it was precisely issues of arbitrary restriction that remained critical in Hungary. If the Téléki government believed that the laws of 1938 and 1939 were satisfactory in restraining Jewish participation, the Germans did not, accusing the Hungarians of not going far enough. Anxious to preserve Hungarian-German relations, in November 1940, Téléki endorsed the Tripartite Pact signed by Germany, Italy and Japan. He then visited Hitler in Vienna. The latter, at that point, was considering sending Europe's Jews to the French colonies, all of which he discussed with Téléki, who apparently agreed that the continent should be free of the Jewish presence.[98] Having aligned herself with the Axis, Hungary was now no longer neutral. Part of her purpose here lay in her revisionist designs to regain territories lost to Yugoslavia after Trianon. By March 1941, however, Hitler had decided to invade Yugoslavia as well as Greece. Téléki conceded on the use of Hungary for passage of German troops to Yugoslavia. This, in turn, brought a threat of reprisal from Britain. Under the intense pressure, at the beginning of April, Téléki committed suicide. The Germans attacked Yugoslavia, and the Hungarians followed through, annexing their old territories, including the Délvidék, in the Yugoslavian northwest. Téléki was replaced by his foreign minister, László Bárdossy, whose tenure would show the 1938-39 bid for stability to be futile, and prove disastrous for the Jews of Hungary.

Conclusion

In his absorbing account of the dialectic between creation and discovery in mathematics, Ulam protégé and M.I.T. combinatorics specialist, Gian-Carlo Rota, describes the field of mathematics as the ultimate escape from reality: "All other escapes . . . are ephemeral by comparison. The mathematician's feeling of triumph, as he forces the world to obey the laws his imagination has

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[98] Braham, *The Politics,* (cit. n.10), p. 177.

freely created, feeds on its own success. The world is permanently changed by the workings of his mind and the certainty that his creations will endure renews his confidence as no other pursuit".[99]

Not even the most meticulous examination of the archival record can provide access to von Neumann's mind. His thought processes as far as the creation of game theory is concerned, the passage between the blank page and the stable-set solution, can never truly be re-discovered. Nonetheless, when one considers the circumstances of the theory's creation and the herculean effort devoted to it, and the manner in which it captures aspects of social organisation then being thrown into sharp relief by history, a certain coherence emerges and one gains a plausible inkling of how he did what he did. Perhaps this is the most that can be expected.

Von Neumann's reaction was unique. His friend, Ortvay, for example, also a mathematical physicist, reacted to the upheaval of the period by probing the functioning of the mind and the formation of attitudes. He was prepared to emphasize complicated psychological forces involving repression, aggression and revenge. Von Neumann chose different emphases: rational selfishness, yet one that existed in the context of extra-rational social norms or "standards of behavior": prejudices or privileges that were there today simply because they had been yesterday. On such attitudes depended equilibrium, and when they changed so did the latter. It is difficult not to see in von Neumann's approach an element of perhaps subconsious resistance to the conditions of the time; an almost defiant willingness to produce a mathematics that would make sense of the world,

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[99] Gian-Carlo Rota, *Indiscrete Thoughts* (Boston: Birkhäuser, 1997), p. 70. Rota goes on to illustrate the "monstrosity" of the mathematician's view of the world by comparing him to none other than Nabokov's Luzhin, "who eventually sees all life as subordinate to the game of chess".

heralding an inevitable return of order, the emergence of a new equilibrium, once the present transition, with its "abnormal spiritual tensions", was over. His perception of the events of the day helped stimulate the mathematics, and the latter, in turn, provided a filter through which he could continue to interpret the world.

His wartime activities culminated in his involvement in the Los Alamos Project, where he not only worked on the mathematics of detonation but was directly involved in the inner circle that, choosing Hiroshima and Nagasaki, oversaw – to use his own term – the "liquidation" of Japan after all.[100] Perhaps he felt that the pathological tensions abroad in the world could be "nudged" towards resolution. By the late Forties, he was noted for his hard line with the U.S.S.R., his bellicose attitude serving to alienate him from friends and acquaintances. Indeed, the episode with which we opened this paper – his abandoning Klari on the Riviera in 1949 – coincided with the beginning of this difficult time.[101] Ulam felt that von Neumann's belligerence here derived from his having too formal and game-theoretic a view of the world.

To the very end, von Neumann continued to speak about game theory in terms congruent with our account. In 1953, young Princeton mathematician Harold Kuhn wrote to him, asking him about the possibility of testing the stable set solution using the experimental methods then beginning at the RAND Corporation. Von Neumann replied in the negative: "I think that nothing smaller than a

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[100] Richard Rhodes, *The Making of the Atomic Bomb*, (New York: Simon & Schuster, 1986), pp. 626ff; Macrae, *John von Neumann* (cit. n. 5), pp. 241-245

[101] Klari von Neumann "Johnny" (cit. n. 2) pp. 32-34.

complete social system will give a reasonable 'empirical' picture [of the stable set solution]. Here, over relatively long periods of time, one can meaningfully assert that the 'system' has not changed, while the positions of various participants within it may have changed many times. This would seem to me to be the analogue of a single solution and an 'exploration' of the imputations that belong to it. After relatively long times, there occur discontinuous changes, 'revolutions' which produce a different 'system'".[102]

Nor should we be surprised that von Neumann was dismissive of John Nash's 1950 proof of the existence of an equilibrium point in a game without coalition formation.[103] To von Neumann, Hungarian Jew and product of Central European society, the formation of social alliances was *sine qua non* in any theory of social organisation. It is easy to understand why the idea of non-cooperation would have appeared artificial to him, elegance of Nash's proof notwithstanding.

The resistance he showed Nash in 1950, von Neumann maintained to the end. At a Princeton conference in 1955, the year he was diagnosed with bone cancer, he defended, against the criticism of Nash himself, the multiplicity of solutions permitted by the stable set: "[T]his result", he said,

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[102] Von Neumann to Harold Kuhn, April 14, 1953, Container 24, File: Kuhn, H.W., Von Neumann Papers, Library of Congress.

[103] This is the contribution for which Nash would be awarded the "Nobel Prize" in economics in 1994. On von Neumann's dismissive attitude towards the Nash Equilibrium, see Robert Leonard, "Reading Cournot, Reading Nash: the creation and stabilization of the Nash equilibrium", *Economic Journal*, May 1994, *104*: 492-511 and Sylvia Nasar, *A Beautiful Mind.*

"was not surprising in view of the correspondingly enormous variety of observed stable social structures; many differing conventions can endure, existing today for no better reason than that they were here yesterday".[104] Within two years, however, von Neumann was gone, and with him the knowledge of what game theory owed to the demise of *Mitteleuropa*.

Coda

Ortvay's letters from Budapest came to a halt in 1941. In January, he was sending three separate copies, to be sure that they reached von Neumann. He appealed for help with colleagues, and with funds for the beleagured Mathematical and Physical Society. He continued to write about the application of mathematics to the realm of "spiritual" states. He was now reading Kurt Lewin's *Principles of Topological Psychology*, and felt it likely that the areas most ripe for such mathematical treatment were those where we made sharp distinctions, such as music, or juridical systems.[105] His last letter, in February, was a brief summary of the previous one, written as though he thought von Neumann had not received it the first time.

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[104] Philip Wolfe, "Report of an informal conference on Recent Developments in the Theory of Games", (Logistics Research Project, Dept. of Mathematics, Princeton University, Jan. 31 – Feb. 1, 1955), p. 25.

[105] Ortvay to von Neumann, January 29, 1941. It should be possible to achieve in these areas, Ortvay felt, what had been achieved in the science of heredity, where natural selection became something that could be discussed rigorously once the essentials were properly treated. Here he continued to describe what he saw as the appropriate way to model the functioning of the neural system. As Aspray, *John von Neumann* (cit. n.45) points out, von Neumann responded to Ortvay's

In 1944, by which time von Neumann and Morgenstern's book had appeared, matters had worsened in Budapest. Because of their services to the State, von Neumann's teachers Fejér and Riesz were granted special status and each allowed to spend part of the war in the one of the protected houses in Budapest's "little ghetto", around Pozsonyi and Szent István streets.[106] These houses were under the diplomatic protection of various countries, and it was here that the Swedish diplomat Raoul Wallenberg managed to save many Hungarian Jews. Fejér and Riesz appear to have been housed in the hospital of the Swedish Embassy at 14 – 16 Tátra St.: Riesz early in 1944, when the Jews of Szeged and the provinces were being deported; Fejér later in the year. There, although crowded, in terrible conditions, with up to fifteen in a room, they were at least safe from deportation, and they survived the war.

So, too, albeit with greater precarity, did their student, Paul Turán. Looking back on the time many years later, on the eve of his death, Turán remembered turning to mathematics. In September 1940, he had been making a living as private tutor in Budapest when he was called to labour camp service. A friend in Shanghai had recently written him about a problem in graph theory: what is the maximum number of edges in a graph with *n* vertices not containing a complete subgraph with *k* vertices? In the camp, Turán was recognized by the commandant, a Hungarian engineer with mathematical training. The commandant took pity on Turán's weak physique and gave him an easy

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suggestions not at this point but in 1955, when he read Warren McCulloch and Walter Pitts, "A Logical Calculus of the Ideas Immanent in Nervous Activity", *Bulletin of Mathematical Biophysics*, *5*: 115-133.

[106] See Frojimovics et al (cit. n.10), pp. 402-403.

job, directing visitors to piles of wooden logs of different sizes. In this "serene setting", Turán recalled, he was able to work on the extremal problem in his head and solve it: "I cannot properly describe my feelings during the next few days. The pleasure of dealing with a quite unusual type of problem, the beauty of it, the gradual nearing of the solution, and finally the complete solution made these days really ecstatic. The feeling of some intellectual freedom and being, to a certain extent, spiritually free of oppression only added to this ecstasy".[107] And who was the camp commandant in question? One Joszéf Winkler, erstwhile contributor to *KöMäL* and joint winner of the Eotvös Competition eighteen years previously - in 1926, none other than the year with König's question about the knight's move on the infinite chessboard.

In July 1944, by which time the threat of deportation was real, Turán was working in a brick factory near Budapest. There, all the kilns were connected by rail to all the storage yards, but, at the crossings, the moving trucks tended to jump the tracks. He began to work on the graph-theoretic problem of minimizing the number of crossings in a yard with *m* kilns and *n* storage yards. This time, however, his thinking was stifled by fears for his family. By late 1944, there was no work to do, but Turán and other Jews expected to be deported from one day to the next. He began to think about another problem, concerning the maximal size of subgraph in a graph of given size. He conjectured a solution, for which he had no support other than "the symmetry and some dim feeling

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[107] Paul Turán, "A Note of Welcome", *Journal of Graph Theory*, 1977, *1*: 7 – 9 on p.8. Questions of the psychological ambiguity and ambivalence of life in confinement are central to the work of Hungarian writer, Imré Kertész, recipient of the Nobel Prize in Literature in 2002. See his *Fateless*, trans. by Christopher C. Wilson and Katharina M. Wilson (Evanston, Ill.: Northwestern University Press, 1992).

of beauty; perhaps the ugly reality was what made me believe in the strong connection of beauty and truth. But this unsuccessful fight gave me strength, hence, when it was necessary, I could act properly".[108]

Others were not so sustained. Dénes König's elder brother, the literary scholar, György, took his life after the German occupation of Budapest on March 19. Then, when the *Nyilas* took over on October 16, König himself, the very one who had introduced von Neumann to the mathematics of chess almost twenty years previously, also committed suicide. Under the Arrow Cross gangs of the *Nyilas*, Hungary entered its darkest period, with Jews being tortured and shot, their bodies dumped into the Danube. At one point, in late December, Fejér and the occupants of the Swedish hospital were marched by night to the river's edge, but saved by the last minute intervention of an army officer.[109] The local attacks, forced labour and deportation to the camps saw the destruction of Hungarian Jewry: 600,000 perished within a few short months. On January 2, 1945, when the Germans were fleeing and the Russians about to enter Budapest, von Neumann's friend Ortvay took his own life, apparently fearing revenge by the "liberators".[110] Neither he, König nor others close to its genesis would get to read the *Theory of Games.*

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[108] *Ibid*, p. 9.

[109] See Turán, "Fejér Lipót, 1880-1959" (cit. n.19), p. 1205.

[110] Dr. László Filep notes that, being unassociated with the Nyilas, Ortvay had no need to fear the Russians, but that the episode illustrates well the fear and tension abroad in Hungary at this time. In a letter to his brother, Marcel, in Sweden, written in July 1945, Frigyes Riesz wrote of the König and Ortvay suicides, and of Fejér's sufferings during the war. He also said that Szeged

mathematician István Lipka (see our photo above) had been fired the previous day from his university position, having been discovered to have joined the Nazi party as early as 1939. (Riesz, F. to M. Riesz, July 18, 1945, Marcel Riesz Papers, Lund, Sweden). I am grateful to Dr. Filep for providing me with a copy of this letter.