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Are price hazard functions really decreasing functions?*

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Abstract

I examine the slope of the price hazard function using microdata for the Brazilian consumer price index. I estimate the price hazard function slope by considering heterogeneity among the items and using a Weibull model with frailty and a finite mixture model (FMM). As in the literature, my results reproduce decreasing hazard functions using a simple Weibull model. However, when I consider heterogeneity among items (using either a Weibull model with frailty or an FMM), the evidence suggests an increasing hazard function. I emphasize that a hazard function may decrease over time if it is composed of heterogeneous hazard functions.

JEL Codes: E31, C41

Keywords: Hazard Function, Heterogeneity, Finite Mixture Model

1 Introduction

Macroeconomic models are used to suggest economic policies and discuss their consequences. Macroeconomics faces the challenge of building models with solid microfoundations that are consistent with the evidence of firms' price adjustments. Price stickiness is fundamental to widely used New Keynesian models. However, it is important to detail the mechanisms behind the price rigidity of individual firms. The probability that a firm changes its price over time is described by a hazard function.

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To understand how firms set their prices, I examine the slope of the hazard function in this work. The empirical evidence mostly indicates decreasing hazard functions over time across countries, methods and databases (Klenow and Malin 2011; Nakamura and Steinsson 2008; Campbell and Eden 2014; Eden and Jaremski 2009; Dhyne et al. 2006; among others). This empirical regularity is considered a puzzle because it does not seem reasonable that the longer firms keep their prices, the fewer opportunities there are for the firm to adjust those prices. However, a hazard function may be decreasing over time if it is composed of heterogeneous hazard functions[1].

As the literature presents evidence of the heterogeneity of hazard functions, I illustrate the effect of aggregation bias due to this heterogeneity following Fougère et al (2007) and Cameron and Trivedi (2005). Consider that the economy has two types of goods with flexible pricing (F) and rigid pricing (R). Assume that F and R have constant hazard rates of 0.4 and 0.1, respectively, and suppose that the economy is composed of 50% F-type goods and 50% R-type goods. Consider a sample of 100 products of type F, of which 40 change in price during the first period, 24 change in price during the second period (60 products whose prices have not changed times 40%) and 14 change in price during the third period. For goods of type R, consider a sample of 100 products, 10 of which change in price during the first period, 9 change in price during the second period (90 products whose prices have not changed times 10%) and 8 change in price during the third period. Thus, the following are the aggregate proportions of price changes (the aggregate hazard rates):

first period $\rightarrow \frac{40+10}{200} = 0.25$

second period $\rightarrow \frac{24+9}{150} = 0.22$

third period $\rightarrow \frac{14+8}{117} = 0.19$

This illustration indicates that the hazard function of the economy is declining over time when two different types of goods that have different, constant hazard rates are aggregated. Thus, unaddressed heterogeneity leads to biased estimates. The probability of observing price changes is lower for products with long spells compared to products whose prices always change[2]. Thus, my point is that the puzzle of a decreasing hazard function stems from aggregation bias. As far as I know, only Cavallo (2015) and Ikeda and Nishioka (2007) have obtained increasing hazard functions[3].

[1] A hazard function may be decreasing in the presence of temporary sales, which requires filtering the data (Nakamura and Steinsson 2013). Klenow and Kryvtsov (2008) estimate a horizontal hazard function for the regular price (except for a peak indicating an increase in the probability of changing the price when the spell reaches 12 months, which is common in studies). Nakamura and Steinsson (2008) estimate a decreasing hazard function for the first few months, which then becomes horizontal at the regular price.

[2] A spell is the duration or amount of time over which a price is maintained. For example, if the water price remained the same from January to March 2005, this spell lasted three months.

[3] Fougère et al (2007) sought to determine whether the hazard function of the economy was decreasing.

This puzzle reflects a methods problem, and this paper presents solid evidence that the hazard functions are not decreasing, as does the work by Ikeda and Nishioka (2007). The finite mixture model (FMM) employed in this work is based on Ikeda and Nishioka (2007), who introduced this methodology in an empirical discussion[4]. The FMM divides the sample into groups (which prevents or reduces aggregation bias) and estimates a hazard function for each group. This method select groups of items with similar probabilities of price changes. Dividing the sample into items with short spells and those with long spells reduces the effect of heterogeneity. This division allows the estimation of a hazard function for each group and may decrease aggregation bias.

This article follows the methodology of Ikeda and Nishioka (2007), with some differences. The most important difference between this and Ikeda and Nishioka's (2007) work is that the quality of the Brazilian data is higher. Ikeda and Nishioka (2007) use the average prices of items at outlets in a Japanese city, i.e., these researchers observe not the price of a soda at a supermarket but the average price of soda at all supermarkets in a particular Japanese city. Thus, the Brazilian data are better because I observe the actual price of a soda at a specific supermarket in the city. But I use the Brazilian data for other reasons as well. The data include the collection of prices for a great share of products every 10 days, allowing for more frequent researches than only monthly. And during our period of analysis, Brazil experienced important macroeconomic variability and inflationary shocks. My sample begins in 1996, the end of one of the most extraordinary disinflationary processes a great economy has passed for decades. In 1997 and 1998 Brazil suffered important effects of the Asian and Russian crises, and at the beginning of 1999 the country moved from a crawling peg exchange rate regime to a floating exchange rate coupled with inflation targeting. After going through blackouts and energy rationing in 2001, the prospects of a left-wing presidential candidate victory generated considerable confidence crisis in 2002 (uncertainty about fiscal and monetary decisions that this candidate could take). The elected government adopted a policy of an orthodox stabilization that resulted in a significant reduction of inflation and macroeconomic uncertainty in 2003. All these episodes produced sizeable variation in inflation, among other variables of the economy.

The results produce decreasing hazard functions for the simple Weibull model, as reported in the literature. However, this article obtains an increasing hazard function for the Weibull model with frailty (already dealing with heterogeneity) unlike Matsuoka (2010), who uses the same methodology and obtains a decreasing hazard function for Japan. However, the slope parameter of the hazard function may be underestimated by the Weibull model compared to the estimate obtained by the FMM (although both

[4]Alvarez et al. (2005) used the same methodology, but they estimated a model based on a simulation of heterogeneous firms with Calvo pricing.

of these models lead to increasing hazard functions). My results indicate an increasing hazard function over time for all three groups of the FMM. Ikeda and Nishioka (2007) notes that some goods and services have increasing functions, whereas this research suggests that all goods and non-tradables have increasing functions. I still divide the sample (into subsamples of goods and non-tradable goods) to estimate the hazard function slopes separately as Ikeda and Nishioka (2007) do. I estimate these using different methodologies, and the results do not differ much when using the entire sample or the subsamples.

The next section addresses the literature review. The third section discusses the methodologies of duration models. The fourth section includes a brief description of the data and what affects the empirical strategy used. The following section presents the results. Then, I offer some final remarks.

2 Literature review

The hazard function of a price change is defined as the conditional probability of the price changing over time. Evidence of decreasing hazard functions have been obtained in several papers (e.g., Klenow and Malin 2011; Nakamura and Steinsson 2008; Campbell and Eden 2014; Eden and Jaremski 2009; Dhyne et al. 2006), presenting a puzzle. If inflation is the only reason for a price adjustment, then a firm's incentive to change its price increases over time because the price moves away from optimal price. This explanation is based on the Ss model developed by Caplin and Spulber (1987). In this case, the hazard function should increase because the probability of a price change increases over time. In the menu cost or state-dependent models (of which the Ss model is an example), transitory, idiosyncratic shocks flatten the hazard function because they lead to temporary price changes that are quickly reversed (Nakamura and Steinsson 2013). In the time-dependent model by Calvo (1983), the probability that a firm adjusts its price is constant, and the hazard function is horizontal because the chances of adjusting the price this month are the same for next month.

The empirically observed decreasing hazard functions may be due to the aggregation of functions for different goods. Other hypothesized mechanisms can also lead to decreasing hazard functions, but this paper will focus on the presence of heterogeneity as an explanation[5]. That heterogeneity generates decreasing functions is a well-known empirical regularity in the survival literature (see Kiefer 1988) and has been considered in the pricing literature more recently. For example, Alvarez et al. (2005) obtained a decreasing hazard function by aggregating groups of agents (with constant hazard

[5]As noted in the introduction, a decreasing hazard function might also be due to temporary sales.

rates like Calvo). In this case, the decreasing hazard function results from aggregation bias.

Thus, unaddressed heterogeneity leads to biased estimates. The probability of observing price changes is lower for products with long spells compared to products whose prices change regularly. One way to address this heterogeneity bias is to randomly select only one spell per product[6]. Ikeda and Nishioka (2007) criticized this procedure because it can bias the estimation of the hazard function. Thus, Ikeda and Nishioka (2007) used multiple spells (the price trajectory) to estimate hazard functions. The authors considered all available information, which should allow for more accurate estimation. However, the use of multiple spells rather than a single random spell per does not lead to different results in the literature. A second way to address heterogeneity is through the FMM methodology. Only Alvarez et al. (2005) and Nishioka and Ikeda (2007) have used this methodology; however, these studies address different concerns. Alvarez et al. (2005) found that heterogeneous firms with Calvo price produce a decreasing hazard function if heterogeneity is not addressed (by the FMM methodology), while Ikeda and Nishioka (2007) found an increasing hazard function using a Japanese database, indicating that the puzzle stems from the use of an inadequate methodology.

As far as I know, only Cavallo (2015) and Ikeda and Nishioka (2007) have clearly obtained increasing hazard functions. In my opinion, the Ikeda and Nishioka (2007) study is more interesting based on its methodology. The different results obtained by Ikeda and Nishioka (2007) are based on changing the methodology rather than the data source. Ikeda and Nishioka (2007) used Consumer Price Index (CPI) data for Japan between 2000 and 2004. By comparison, Higo and Saita (2007) and Matsuoka (2010) obtained decreasing hazard functions for Japan using the same database. Ikeda and Nishioka (2007) estimated a decreasing hazard function using a methodology similar to that of Higo and Saita (2007). However, Ikeda and Nishioka (2007) obtained increasing hazard functions by addressing unobserved heterogeneity through the FMM methodology. Matsuoka (2010) used a Weibull model with frailty, considering individual, unobserved heterogeneity differently than do Ikeda and Nishioka (2007). As far as I know, Matusoka (2010) is the only article that has used a Weibull model with frailty to estimate the hazard function, but this author also obtained decreasing hazard function.

Cavallo (2015) also obtained an increasing hazard function using scraped data for Brazil, Argentina, Chile and Colombia for 80,000 products between 2007 and 2010[7]. This article obtained increasing hazard functions using a simple, non-parametric

[6]Studies that consider only one spell for each randomly selected item include Alvarez et al. (2005), Dias et al. (2007), and Aucremanne and Dhyne (2005).

[7]The database collects online information about the prices of goods.

methodology and excluding temporary sales[8]. . However, Cavallo (2015) arbitrarily divided the sample into subgroups according to the average duration of the price spells to reduce aggregation bias. Thus, the author estimates a hazard function for each subgroup. Basically, Cavallo (2015) created subgroups arbitrarily, while Ikeda and Nishioka (2007) estimated the model and identified subgroups.

Fougère et al. (2007) were already concerned with the evidence that these hazard functions are not decreasing. They wanted to prevent biased estimates of the hazard function caused by inappropriate treatment of heterogeneity. They used disaggregated data divided into groups (by type of good and outlet) to consider heterogeneity. The authors found that a constant hazard function could not be rejected by 35% of the weighted CPI in France. Additionally, this article indicated that the assumption of non-decreasing hazard functions (constant and/or increasing) could not be rejected in 75% of cases. Finally, they distinguished between increasing and decreasing price spells.

Dias et al. (2007) were aware that heterogeneity affects the hazard function slope. Thus, they used a different methodology from other authors, assuming no ex ante functional form for the hazard function. Using a discrete time model and Portuguese data, where the dependent variable is binary (either the firm changes the price of the item or not), the authors saturate the estimation by including a dummy variable for each spell duration. This procedure allowed them to estimate a coefficient for each price duration in a flexible manner. Thus, this method allowed them to capture the effects of covariates without making additional assumptions about the distribution of any neglected individual heterogeneity. However, the authors were interested in estimating a hazard function and determining whether pricing was dependent on the state or time. Therefore, the authors were concerned with assessing the effects of covariates (cumulative sectoral inflation and output growth). Dias et al. (2007) obtained a decreasing hazard function, but they saturated the regression to produce accurate estimates of the covariates (rather than an accurate slope of the hazard function). In other words, the authors used an empirical strategy to mask the effect of heterogeneity, with a distinct purpose from that in the present work. I discuss the duration model methods that I use in this work below.

3 Duration models methods

This section presents the three methodologies used. The first method is a Weibull

[8] The author used the Kaplan and Meier (1958) method for the hazard function, which is the proportion of items that have changed in price at time t of all the items that started period t without changing its price.

duration model, followed by a model with individual unobserved heterogeneity (frailty). The third method uses FMMs to analyze the durations of spells.

I use a Weibull model because these are traditionally used in duration models and survival analysis. The disadvantage of this model is that it is not flexible, as this model does not allow variation in the hazard rate over time (for example, a parabolic hazard rate). However, this model is interesting for pricing because it is possible to determine whether the hazard function is horizontal, increasing or decreasing based on a single estimated parameter.

The Weibull model is specified as

$$\theta(t, x) = \alpha\lambda t^{\alpha-1} = \alpha exp(x\beta)t^{\alpha-1} \tag{1}$$

where $\lambda = exp(x\beta)$, $\alpha > 0$, θ is the hazard rate, and x is the covariate vector. The hazard rate $\theta(t, x)$ is the probability that a firm changes the price of the good in time t conditional the firm keep the price until time t-1. The hazard rate increases monotonically over time t if $\alpha > 1$ and decreases monotonically over time t if $\alpha < 1$. If $\alpha = 1$, the hazard rate is constant over time[9]. That is, the shape of the hazard function depends on α, whereas λ simply changes the hazard function scale.

The estimation is based on the maximization of the likelihood function (2) for all i observations. The likelihood function calculates the density of the i-th observation as

$$f\left(t_i|x_i, \alpha, \beta\right)^{\delta_i} S\left(t_i|x_i, \alpha, \beta\right)^{1-\delta_i} \tag{2}$$

where $\delta_i = 1$ if the item i has changed the price, and $\delta_i = 0$ if the price item i has not changed. The density of the i-th observation is calculated by the probability $f\left(t_i|x_i, \alpha, \beta\right)$of a change in the price for items that have changed in price or the survival function $S\left(t_i|x_i, \alpha, \beta\right)$ if the item i has not changed in price. The probability $f\left(t_i|x_i, \alpha, \beta\right)$of a price changes at time t is equal to the hazard rate $\theta(t, x)$ oo items that "survived" until moment t $S\left(t_i|x_i, \alpha, \beta\right)$, or

$$f\left(t_i|x_i, \alpha, \beta\right) = \theta(t, x)S\left(t_i|x_i, \alpha, \beta\right) \tag{3}$$

The survival function $S\left(t_i|x_i, \alpha, \beta\right)$ is calculated for those items that have not "died" (i.e., have not changed price), or $S\left(t_i|x_i, \alpha, \beta\right) = 1 - F\left(t_i|x_i, \alpha, \beta\right)$, where $F\left(t_i|x_i, \alpha, \beta\right)$ is the cumulative density function of the items that have changed price following the Weibull distribution given by $F\left(t_i|x_i, \alpha, \beta\right) = 1 - exp\left(-\lambda t^{\alpha}\right)$. In this way, I maximize the logarithm of likelihood function (2) to estimate the required coefficients considering

[9] When you reject the null hypothesis that$\hat{\alpha} = 1$, it is not possible to simplify the Weibull model into an exponential model (Jenkins 2005).

all individuals (Jenkins 2005)[10].

The Weibull model shown in equation(1) disregards the presence of unobserved heterogeneity, so any differences among individuals are captured only by the observed explanatory variables x. The next step is to allow the presence of unobserved individual effects, which is called frailty survival analysis. To the best of my knowledge, only Matsuoka (2010) has used this model to analyze price duration. If the effects of unobserved heterogeneity are important and are ignored in modeling, the response of the hazard rate in relation to the covariates will be underestimated. In the presence of heterogeneity, the hazard rate $\theta(t,x|v) = v\theta(t,x)$ depends on the observable characteristics x and v, which is an unobservable individual effect. The random variable v has the properties $v > 0$, $E(v) = 1$ and variance $\sigma^2 > 0$, and this variable is distributed independently of x and t. The likelihood ratio (LR)test checks for the presence of individual unobserved heterogeneity. Thus, this test has $\sigma^2 = 0$ (i.e., the absence of individual heterogeneity because v would be fixed for all individuals) as the null hypothesis. Under the assumptions of a Weibull model, the hazard rate with frailty can be written as $\theta(t,x|v) = v\alpha exp(x\beta)t^{\alpha-1}$. Firms with values $v > 1$ have higher hazard rates than does the median firm, and their chances of maintaining their prices are lower. Basically, v is an additive "error" term, a random intercept when the equation is in logarithmic form. Furthermore, the individual effects are too large to be estimated, leading to insufficient degrees of freedom for those estimates. The solution is to assume that v has a distribution with a functional form that is summarized by a few parameters so that it is possible to estimate these parameters (Cameron and Trivedi 2005). In this work, we assume that v has an inverse Gaussian distribution to ensure the estimation of the Weibull model with frailty.

Finally, I present the FMM. On the one hand, FMMs provide a natural way to address heterogeneity because they divide a sample into a finite number of types or groups, which is a flexible and parsimonious approach. The FMM considers that the population consists of homogeneous subpopulations. On the other hand, the previous methodology (the Weibull model with frailty) assumes a continuous distribution for the unobserved heterogeneity and estimates the parameters of this distribution assuming a homogeneous population. In other words, the FMM is a fixed effects model with random groups, and the Weibull model with frailty is a random effects model with known groups; see Cameron and Trivedi (2005) for more details.

In the FMM, the sample arises from a population consisting of a finite number of latent classes, types or groups. That is, each element of the sample is obtained from such sub-populations. Consider $f(y_j)$, the density of the random variable observed

[10] The likelihood function is given by $L = \prod_{i=1}^{N} f(t_i|x_i,\alpha,\beta)^{\delta_i} S(t_i|x_i,\alpha,\beta)^{1-\delta_i}$ for a total of N individuals.

y_j. Then, I can write that

$$f(y_j) = \sum_{i=1}^{g} \pi_i f_i(y_j) \tag{4}$$

where $f(y_j)$ can be understood as a weighted average of the densities of g subpopulations (or g components), $f_i(y_j)$ is the density of the random variable observed y_j at the i-th subpopulation (or mixture density), and π_i the mixture weights (marginal probabilities) (Deb and Trivedi 1997). That is, the heterogeneous population consists of g groups of size proportional to π_i. The properties of the weights are $0 \leq \pi_i \leq 1$ and $\sum_{i=1}^{g} \pi_i = 1$. The estimation of the coefficients is based on the maximization of the log-likelihood function. However, the estimation is difficult because two sets of parameters must be calculated. As the density functions $f_i(y_j)$ have unknown parameters Ω_i to be estimated, I can write $f_i(y_j; \Omega_i)$. In addition, the unknown weights π_i need to be estimated. Thus, an iterative expectation maximization (EM) approach is required to obtain the unknown parameters (Ω, π) in two steps, where $\Omega = (\Omega_1, ..., \Omega_g)$ and $\pi = (\pi_1, ..., \pi_g)$ (McLachlan, Peel, 2000).

Before I address the EM approach, I consider a dummy variable d_{ij} that identifies whether an observation belongs to a particular group. For each observation j, $d_{ij} = 1$ if this observation is part of the i-th group, and $d_{i'j} = 0$ for the other groups i'. Thus, the EM algorithm consists of iterate steps (1) expectation and (2) maximization until the parameters converge. The first step is the expectation (E) step in which for a given parameter Ω, $\hat{\pi}_{ij}$ and $\hat{\pi}_i$ are estimated using the following equation

$$\hat{\pi}_{ij} = \frac{\pi_i f_i(y_j; \Omega_i)}{\sum_{i=1}^{g} \pi_i f_i(y_j; \Omega_i)} \tag{5}$$

where $\hat{\pi}_{ij}$ is the a posteriori probability of y_j belonging to component i, and $\hat{\pi}_{ij}$ is the estimated expected value of d_{ij} in step E: $\hat{\pi}_{ij} = E(d_{ij})$. In addition, $\hat{\pi}_i = \frac{1}{n} \sum_{j=1}^{n} \hat{\pi}_{ij}$, which means that the average value of $\hat{\pi}_{ij}$ over all j individuals $(\hat{\pi}_i)$ is the probability that a randomly selected individual belongs to subpopulation j. In the maximization step, the log-likelihood function (4) is maximized in relation to Ω given the values of $\hat{\pi}_{ij}$ and $\hat{\pi}_i$ (Alvarez et al, 2005).

The process starts with an initial guess of (Ω, π) for each item (a good in a particular supermarket in the city being considered) and each subgroup i for which $\hat{\pi}_{ij}$ is subsequently calculated. Thus, the iteration of the EM algorithm continues until the estimated parameters $\left(\hat{\Omega}, \hat{\pi}\right)$ converge (Cameron and Trivedi, 2013). In this paper, I assume $f_i(y_j; \Omega_i)$ equals (2), where $f_i(t_i|x_i, \alpha, \beta)$ and $S_i(t_i|x_i, \alpha, \beta)$ have the Weibull form for each component. Thus, the parameters to be estimated for each subpopulation are α and β.

I use the Akaike information criterion (AIC) and Schwarz information criterion (BIC) to indicate whether more groups are preferable. These information criteria penalize the log-likelihood in models with many parameters. The model with the lowest information criterion value is preferred. In the next section, I discuss the characteristics of the data and the empirical strategy used.

4 Data and Empirical Strategy

The official inflation index (IPCA) is produced by the Brazilian Institute of Geography and Statistics (IBGE) based on the inflation target of the Central Bank of Brazil. The Getulio Vargas Foundation (FGV) produces alternative inflation indexes. I use data for individual price quotes for products collected by the FGV. Price quotes for approximately 180,000 different items are collected from 2500 outlets (i.e., at the store level). The sample covers 85% of the CPI between 1996 and 2005. These data were presented and analyzed in Gouvea (2007).

Data for some products are collected every ten days, while others are collected monthly. The price data are collected in the 12 largest cities in Brazil. The sample includes only two cities (Rio de Janeiro and São Paulo) until 2000. Since 2001, ten cities (Belém, Belo Horizonte, Brasília, Curitiba, Fortaleza, Goiânia, Porto Alegre, Recife, Salvador and Florianópolis) have been added to the CPI sample, bringing the total to 12 cities (Barros et al. 2009). The sample comprises 7 million observations before any filtering.

Following the literature, products that are regulated are excluded from the sample. These products account for approximately 30% of the CPI, a considerable portion of the sample. As the pricing of these products is governed by pre-established contracts, firm pricing policies cannot react to shocks instantly. I also exclude outliers from the sample, which were defined as price increases of greater than 900% and price reductions of 90% or more. Such magnitudes can be considered typographical errors. Few observations were excluded as outliers.

The price duration is measure as the number of days over which the price remained fixed. I use multiple price spells for the same item, following Ikeda and Nishioka (2007). Random sampling may lead to bias in the estimation of the hazard functions because each item is characterized by only one randomly selected spell[11]. In this paper, a spell is not only recorded when the firm changes its price but also when I observe the firm's price data (even if the firm does not change the price). I define a

[11]Random sampling does not have serious problems with a large sample if the true hazard functions are exponential, as in Alvarez et al (2005). However, I may lose important information with random sampling if I use random sampling with non-flat functions such as those that I and Ikeda and Nishioka (2007) consider.

minimum number of observations per item in the sample to ensure that some items in the sample are significant. An item was kept in the sample if it had at least 80 observations. Furthermore, spells that were left censored were excluded. After this filtering, a sample of approximately 6.5 million observations remained.

The empirical strategy includes three types of regressions: (i) the Weibull model, (ii) the Weibull model with frailty and (iii) the FMM. The selected specification is composed of dummy variables for the year and city in which the data were collected. The specificity of the data (because the information collected from other cities adds up over time) required me to restrict the city dummy variables such that only the dummy variable for the city of Rio de Janeiro is included. This change was necessary because the FMM does not converge when more city variables are included, as the set of cities is not the same from the beginning of the sample (which first included only two cities). Thus, the same specification is used for the duration models and the FMM to allow for direct comparison of the results. This specification includes dummy variables for the city of Rio de Janeiro and for the years from 1996 to 2004 as well as the constant.

The next step is to classify the items into categories to facilitate the interpretation of the result, especially the FMM estimates. More broadly, I address the division between tradable (composed of industrial goods and commodities) and non-tradable goods. Non-tradable goods include (before eliminating regulated items) communication services, education, food away from home, housing (rents), domestic services, recreation and culture, health care, medical and laboratory expenses and public transportation. The non-tradable category consists mostly of services. The commodities category includes general food items. The industrial category combines cleaning goods, hygiene and beauty products, furniture and decorations, housing appliances, petrol, vehicles, home textiles, telephones and electronic goods, tobacco, beverages and pharmaceuticals goods. For the commodities category, prices are collected every ten days, on average, with a higher frequency of collection than other categories (which are collected monthly, for example). Commodities prices are collected more frequently due to their higher frequency price changes. Feltrin and Guimarães (2015) presented evidence of this greater flexibility in commodity prices due to exchange rate shocks using the same data considered in this study.

An important characteristic of the data used in my work is that effective spells (those that resulted in price changes or in which an item exits the sample) produce 2,897,497 observations, representing approximately 45% of all observed prices. The goods and non-tradables categories represent 96% and 4%, respectively, of the effective spells sample[12]. As previously noted, the same of effective spells includes mostly

[12]Commodities, non-tradables and industrial goods categories present 72.8%, 4% and 23.2% of the sample, respectively. However, non-tradables represent a mere 8% of the sample if I consider the entire sample (rather than only the effective spells).

commodities, i.e., tradable goods.

Regarding the empirical strategy of this work, I do not identify a subgroup of the sample ex ante to estimate an FMM for each subgroup following Ikeda and Nishioka (2007). The two groups that Ikeda and Nishioka (2007) identify ex ante are services and goods. Thus, they are able to obtain subgroups of goods and other subgroups of services. In other words, they estimate an FMM for the goods category and another FMM for the services category. In this paper, I want to let the data describe the behavior of the hazard function without restrictions. Thus, I estimate an FMM for the full sample, allowing certain estimated subgroup to include a mix of goods and services -- which was not allowed by Ikeda and Nishioka's (2007) strategy -- although they are goods of a different nature. Then, I compare the results following the strategy of Ikeda and Nishioka (2007). I present the results in the next section.

5 Results

The simple Weibull duration model results are presented first, followed by those for individual unobserved heterogeneity (frailty) and then the FMM. The focus of this paper is the estimation of the hazard function slope, so I do not present the estimated covariates coefficients. First, I discuss the results of the simple Weibull model. Table 1 presents the regression results for the Weibull duration models without and with frailty. The estimate produced by the simple Weibull model (without frailty) is $\hat{\alpha} = 0.66$, so the hazard function is decreasing over time (because $\hat{\alpha} < 1$), which is consistent with most of the pricing literature [13]. A significant parameter $\hat{\alpha}$ indicates the rejection of the null hypothesis that $\hat{\alpha} = 1$. In this case, there is a direct interpretation: if $\hat{\alpha}$ is significant, then the hazard function is increasing or decreasing but is not constant. This result rules out the economy being conducted by firms that homogeneously price à la Calvo[14]. The interpretation of the hazard function slope is based on the ratio of the hazard rate at survival time t compared to the survival time at u, given by $\left(\frac{t}{u}\right)^{\hat{\alpha}-1}$. Consider survival at time 60 compared to survival at time 30 (where time is the number of days), then the calculation yields $\left(\frac{60}{30}\right)^{0.66-1} = 0.79$. The hazard rate in the second

[13]The specification that includes dummy variables for the city of Rio de Janeiro and for the years from 1996 to 2004 produces slightly different results. For example, the estimated hazard function slope is$\hat{\alpha} = 0.72$ when all year and city dummy variables are included in the simple Weibull model, i.e., the estimated hazard function slope is robust to the inclusion of other covariates. Because the FMM methodology does not allow dummy variables for all cities, I keep only the dummy variable for Rio de Janeiro.

[14]In the case of a homogeneous population of firms with Calvo pricing, the hazard function would be constant. However, Alvarez et al (2005) find that heterogeneous groups of firms with Calvo pricing (the only difference is the probability of changing the price in each period) produce a decreasing hazard function due to aggregation bias.

month of survival is 79% of the hazard rate in the first month (30 days of a constant price).

	Weibull Model		Weibull Model with Frailty	
Alpha	0.66	***	1.23	***
	(0.0003)		(0.0005)	
			Statistics	
LR Test			4400000	***

Table 1: The results for the Weibull models

The next step is to address the Weibull model with frailty (individual unobserved heterogeneity), as shown in Table1. The LR test rejects that the individual unobserved heterogeneity variance is zero, which means that the model with frailty is preferable to the simple Weibull model. The estimated hazard function slope is $\hat{\alpha} = 1.23$. As $\hat{\alpha} > 1$, I obtain an increasing hazard function. This means that the probability that a firm changes its price increases over time. However, Matsuoka (2010) used this methodology for Japan and obtained a decreasing hazard function. This difference in the results is due to differences in the data because the methodology is the same. However, I can estimate the hazard function slope with greater accuracy using a methodology that addresses unobserved heterogeneity in a more appropriate way, such as the FMM methodology. I will present the results of the FMM shortly.

Considering the hypothesis that the decreasing hazard function comes from heterogeneity in the data and that the methodology cannot address this point, an alternative is to estimate the hazard functions for goods and non-tradables separately. The results of the Weibull models (with and without frailty) after dividing the sample into goods and non-tradables are presented in Table A.1 of the Appendix. Considering the simple Weibull model, the estimates for the hazard function slope are 0.73 and 0.56 for goods and non-tradables, respectively. However, these estimates increase to 1.23 and 1.13 for goods and non-tradables, respectively, if I consider the Weibull model with frailty. Dividing the sample into two categories does not change the results substantially because the estimated hazard function slopes for the subsamples are close to the slope estimated for the entire sample by the same method[15].

I discuss the FMM that identifies homogeneous groups (components) to estimate a Weibull model for each component below. My intention to provide an estimated

[15]The estimate for the entire sample is $\hat{\alpha} = 1.228$ for the Weibull model with frailty, while the estimate for the subsample of goods is $\hat{\alpha} = 1.233$. That is, the estimate for the entire sample is within the range generated by the two disaggregated estimates.

hazard function slope without the previous restrictions or divisions of the data. The information criteria for models with 1, 2 and 3 components listed in Table 2indicate that the model with 3 components improves the estimation[16]. Thus, I discuss the FMM model with 3 components.

	Number of components		
	1	2	3
Log-Likelihood	-33502664	-31900644	-31703081
Number of observations	6462185	6462185	6462185
Number of parameters	11	25	38
AIC	67099064	63830224	63426516
BIC	67099194	63830520	63426966

Table 2: Indicating the number of components in Finite Mixture Model

Next, I present the estimates of the Weibull model with FMM in Table 3. The sample is divided into models with one, two and three components with approximate percentages (π) of 52, 34 and 14, respectively. The corresponding hazard function slopes are 3.17, 2.04 and 1.25, respectively. These hazard function slopes are statistically significant, that is, I reject the null hypothesis that the hazard function slope is constant over time (à la Calvo pricing) for any of the components. Thus, I have an increasing hazard function over time for all three components. Ikeda and Nishioka (2007) reported that 68% of goods and 56% of non-tradables were characterized by increasing hazard functions. In my case, the shares are different.

Variables	Component 1	Component 2	Component 3
Alpha	3.17 ***	2.04 ***	1.25 ***
	(0.0006)	(0.0058)	(0.0015)
Pi	52.01	33.57	14.42
	(0.0012)	(0.0005)	(0.0005)

Table 3: The results for the Finite Mixture Model

If I consider the weight of each component, the slope of the aggregate hazard function is $\hat{\alpha} = 2.51$. The estimated hazard function slope for the FMM ($\hat{\alpha} = 2.51$) is higher than that obtained for the Weibull model with frailty ($\hat{\alpha} = 1.23$). Thus, my

[16]The model with one component is merely the simple Weibull model shown in Table 1.

empirical evidence for Brazil indicates that the estimated hazard function is increasing for the entire sample (all components). This means that the probability that a firm changes its prices is increasing over time. To illustrate, the hazard rate at the end of the second month is 286% of the hazard rate at the end of the first month (if the firm has not changed its price during the first month). The firm has a higher chance, by 186 percentage points, of changing the price of this good in the 2nd month if it maintained that price for one month.

Then, I detail those componentsin Table 4. One point that illustrates the differences among the three components is the average expected length of the spell for each component. The average spells for components 1, 2 and 3 are 16, 61 and 266 days, respectively. Recall that the sample is comprised of intervals over which I observe the price. In this paper, I not only identify a spell when the firm changes its price but also when I observe the firm's price data (even if the firm has not changed the price). Basically, the first component contains short spells, while the third component covers long spells. As the spell is very short (and the hazard function is strongly increasing), the first component can be called the "flexible pricing group"[17]. The first component is dominated by goods, especially commodity goods for which we have 10 observations over 10 days (representing 80\% of the first component)[18]. . As the spell is long, the third component can be called the "sticky price group". Many non-tradable items are included in the third component (sticky price group), which includes long spells and increasing hazard functions (but with smaller slopes). There is evidence that services have more rigid prices both around the world (Bils and Klenow 2004; Nakamura and Steinsson 2013) and in Brazil (Gouvea 2007; Barros et al. 2009; Feltrin and Guimaraes 2015). There are a few explanations for this phenomenon. The lower frequency of price adjustment for services may be due to lower volatility of consumer demand for such goods (Bils and Klenow 2004) or may be indirect evidence of wage rigidity, as the costs of services is more closely linked to wages (Nakamura and Steinsson 2013).

			Categories		Average length of
		Share	Goods	Non-Tradables	spells
Components	**1**	52	97	3	16
	2	34	92	8	61
	3	14	76	24	266

Table 4: Descriptive Statistics of the Finite Mixture Model Components

[17] The average interval observed between prices of the same item is 16 days, and as the hazard function is strongly increasing, it is likely that the firm changes its price in a short time.

[18] I present the same Table 4, breaking down the categories of goods into commodities and industrialized goods, in Table A.2 of the Appendix to help clarify the intuition for the results.

Thus far, the strategy for estimating the FMM was to let the data to speak about the hazard function slope without restrictions. Another way to estimate the hazard aggregate function is to divide the sample into two categories (goods and non-tradables) and estimate an FMM for each sample. In this case, I am intervening by dividing the sample into categories. This strategy is similar to that used by Ikeda and Nishioka (2007). I present the results of the model with two components for each of the two subsamples in Table A.3 of the Appendix[19]. In the case of goods, the estimated hazard function slope is 2.59 for 65% of the sample and 1.17 for the remaining 35%. For non-tradables, the slope is 2.21 for 30% of the sample and 1.08 for the remaining 70%. That is, even when adopting the same strategy used by Ikeda and Nishioka (2007) to divide the sample ex ante into goods and non-tradables, our results indicate an increasing hazard function for all components, unlike the results obtained Ikeda and Nishioka (2007). The weighted hazard function slopes are 2.09 for goods and 1.42 for non-tradables. If I want to obtain the slope of the aggregate hazard function from the FMM estimates for the subsamples of goods and non-tradables, I can use the weights, which lead to an estimated slope of 2.04[20]. This estimated aggregate hazard function slope (2.04) is not very different from that obtained by FMM for the entire sample (2.51). Regardless, dividing the sample into groups containing goods and non-tradables does substantially change the increasing hazard function obtained for the entire sample.

Final remarks

The microeconomic pricing literature reports mostly decreasing hazard functions for different countries, methodologies and data sources. This empirical regularity is considered a puzzle because it seems unreasonable that the longer firms maintain their prices the lower the opportunities for the firm to adjust those prices. The aim of this paper is to determine whether the hazard function is decreasing over time when it is composed of heterogeneous hazard functions.

In my work, the Weibull model, which does not address heterogeneity, leads to a decreasing hazard function, as in most of the literature. When I consider the heterogeneity among items (the Weibull model with frailty or the FMM), the hazard function is increasing over time. In the case of the FMM, the sample is divided into three parts

[19]The information criteria indicate that the model with two components is preferred (relative to the model with one component) for both subsamples.

[20]The weights were based on the number of observations, which leads to 92% for goods and 8% for non-tradables.

or components. These three components include two clearly opposing groups. The first component -- representing one-half of the sample -- includes items with short spells and strongly increasing hazard functions. Most of the items in the services category are included in the third component (the “sticky price group”’), which has long spells and increasing hazard functions (but the smaller slope among the components). As a robustness check, I divide the sample and estimate these model for each subsample, and the results do not change substantially.

This paper presents evidence that using a methodology that addresses heterogeneity yields increasing hazard functions for Brazil, as in Ikeda and Nishioka (2007) and Cavallo (2015). However, this increasing hazard function does not necessarily imply that pricing is state dependent. This work, as well as that of Ikeda and Nishioka (2007), focuses on building microfoundations for the slope of the hazard function and the presence of heterogeneity.

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Appendix

	Only Goods			
	Weibull Model		Weibull Model with Frailty	
Alpha	0.73	***	1.23	***
	(0.0003)		(0.0006)	
			Statistics	
LR Test			3500000	***

	Only Non-Tradables			
	Weibull Model		Weibull Model with Frailty	
Alpha	0.56	***	1.13	***
	(0.0014)		(0.0027)	
			Statistics	
LR Test			220000	***

Table A.1: Results for the Weibull models after splitting the sample into goods and services

			Categories			Average length
		Share	Commodities	Non-Tradables	Industrialized	of spells
Components	**1**	52	80	3	17	16
	2	34	64	8	28	61
	3	14	49	24	27	266

Table A.2: Descriptive Statistics Alternative of the Finite Mixture Model Components

	Only Goods			
Variables	Component 1		Component 2	
Alpha	2.59	***	1.17	***
	(0.0029)		(0.0010)	
Pi	65.01		34.99	
	(0.0005)		(0.0005)	
	Only Non-Tradables			
Alpha	2.21	***	1.08	***
	(0.0289)		(0.0031)	
Pi	30.31		69.69	
	(0.0032)		(0.0032)	

Table A.3: The results of Finite Mixture Model for the data disaggregated